Stiff Sources and Numerical Methods for Conservation Laws

organized by
Doron Levy and Benoit Perthame

Workshop Summary

The main goals of this workshop were:

1. to identify important applications for balance laws models, identify open problems and to define the directions for future research in this field in terms of what are the more important problems that should be investigated. 2. to exchange ideas on recent advances in numerical methods for balance laws, to compare different methods, to identify their weaknesses, and to discuss possible solutions to these problems.

The workshop was organized as to carefully balance between presentations, discussions, and collaborations between groups.

One can summarize the outcomes from several points of view: (1) specific models arising from applications, (2) solvers and high order accuracy, (3) well balancing for steady states and (4) multiscale relaxation problems.

1. Specific models arising from applications

Throughout the workshop several applications have been presented by specialists coming from physics or engineering departments as well as specialists with a direct link with biology departments and semiconductor applications. Although planed for a large class of models and applications, it appears that the Saint-Venant system for shallow water has been used by a majority of speakers as a motivation (the same is also true for the discussions on numerical methods).

The main conclusions are certainly that for semiconductors physics, environmental physics and engineering, the models (Partial Differential Equations, their coupling and the relations in between) and the numerical difficulties are well established. The participants seemed to agree that the forcing terms in the Saint-Venant system are well understood. The numerical methods for solving this system are well developed. The models that were identified by the participants as the most important directions for future research in this field are: 1. Multilayer Saint-Venant systems (the coupling between the layers is the source of difficulty). 2. The interaction between slow waves and fast waves (e.g. in flows with rotations). 3. In oil recovery, a field that will certainly win interest in the next years because of the applicative outcomes, the difficulties are also well established: solvers for elaborate (non explicit) fluxes, resonances (weak hyperbolicity), coupling with the external velocity arising from the Darcy law for the fluid in a porous media and in a longer range abnormal viscosities. 4. In biological applications, the researchers involved in collaborations are pointing out a different level of difficulties; the models are not well established and questions are even not always precise enough to settle a universal model. The numerical solution of the resulting models is then supposed to provide a qualitative answer.

2. Solvers and high order accuracy

The question has been addressed by several speakers. Several discussions have shown that these questions are no longer a leading issues. By now, a variety of high-order numerical methods exist for approximating solutions to balance laws. While there are still some disagreements as of which methods are better (more efficient and more accurate), it is clear that there is no ultimate numerical method that can qualify as the best in all cases. Relaxation schemes provide a method allowing to solve a wide range of problems with well settled weak points and advantages (especially with fluxes that are too complicated to the point that eigenelements are difficult/impossible to compute). Comparisons between finite volumes and discontinuous Galerkin methods are also well established and several specialists could share their own experience and conclusions.

3. Well-balancing for steady states

Here again, the concept is clearly widely understood in terms of a numerical recipe. All the groups that want to compute with sources have found ways (it is more or less universal to use the Upwinding of the Sources at interfaces in order to well-balance, exactly or not, the uxes and sources.)

It seems that a general understanding of the method is however not so well settled, especially in the context of following issue. But also and more deeply the difficulty arises because the source can generate resonances, i.e., considered as an extended system where the external data B(x) (bottom data for instance) is considered as satisfying d/dt B = 0, two eigenvalues of this new hyperbolic system coincide.

4. Multiscale relaxation problems

Several examples were given of physical problems where one has to consider relaxation terms. This means that a parameter can range from small to large depending on the local medium or another computed quantity. The method of Upwinding of the Sources at interfaces can also be used here. The question is not to achieve a non trivial steady state. In this context, it provides a way to achieve accuracy for the entire range of the parameter. Time multiscale analysis is commonly used by physicists/engineers to find out reduced models and numerical methods should be found to perform this automatically.