Recent Trends in Additive Combinatorics

organized by
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Workshop Summary

One of the main goals of the meeting is to form a community around the fast developing field of additive combinatorics. We are very happy to report that this goal has been accomplished with success beyond our expectations. Researchers from several fields came to the workshop and left with the feeling that a new, unified, direction of research has been created. Immediately after the conclusion of the workshop, two researchers (Green and Granville) proposed new meetings of this type in the next two years (2005 in Bristol and 2006 in Montreal). Our hope is that we will be able to build up a tradition in this area.

The main focus of the workshop was around the following topics:

(1) Techniques which arose from different proofs of Szemeredi's theorem (hypergraph lemma, quadratic Fourier analysis) and where these techniques would be applied: Gowers, Green and Nagle gave excellent talks on these issues. Green shown that quadratic Fourier analysis plays an essential role on his famous work (with Tao) on long arithmetic progressions in primes. This tool may lead to a concrete estimate on the number of progressions of length four in the set of primes between one and n, a task which seemed almost impossible from other methods from analytic number theory. Gowers and Nagle gave talks about the development of the hypergraph version of Szemeredi's regularity lemma, due to Gowers and a group of researchers led by Rodl. This result has several fundamental consequences, among others a new (and perhaps the "simplest") proof of Szemeredi's theorem and also its generalizations due to Furstenberg et al. Nagle also gave an extra lesson touching the technical issues in the proof.

(2) Freiman inverse theorems and the structure of sumsets: Freiman gave an overview about his inverse theorems and introduced several applications, most of which have not been seen by the participants. Sudakov next gave talk about an application (due to Sudakov, Szemeredi and Vu) of the inverse techniques to an old question of Erdos and Moser concerning sum-free sets. Szemeredi gave a survey about his joint work with Vu on long arithmetic progressions in sumsets. These results complement earlier results by various researchers including Bourgain, Freiman-Halberstam-Ruzsa, Green, Sarkozi and Lev and led to several accurate estimates. As corollaries, they confirm several long standing conjectures due to Erdos and Folkman on complete sequences. A sumset problem of a somewhat different nature, which arose in connection with Erdos distinct distances problem was discussed by Tardos. Katz and Tardos showed that tools from probability (such as entropy) are useful in the attack of this problem.

(3) Sum-product problems and exponential sums: Bourgain and Chang gave two inspiring talks about how combinatorial results and ideas which have been developed concerning Erdos-Szemeredi sum-product problem and its finite field variant (due to Bourgain, Katz and Tao) can be applied, combined with other tools, to obtain deep results on exponential

sums and led to breakthrough applications in several areas. A series of results (obtained by Bourgain, Chang, Konyagin and few other co-authors) gave impressive extensions of classical results such as Weil's and Mordell's inequalities. It is very likely that these results will have a long lasting impact.

(4) Incidence geometry and Kekaya's problem and Erdos distinct distances problem: Solymosi gave a survey about problems in incidence geometry and their connection to many well known and hard problems such as Kekaya's problem, Erdos distinct distances problem and the Bomberi Granville-Pintz problem about squares in arithmetic progressions. Laba discussed several variants of Erdos distinct distances problems which arose from problems in analysis and Katz gave an intuitive talk about his combinatorial approach (with Tao) toward to Kakeya conjecture. It is very interesting to see the interplay between combinatorics, additive number theory (especially sumset problems) and geometry in this area. This is also the topic which leads to the most open questions.

The most enjoyable part of the workshop is perhaps the problem section as it brings together different schools of thoughts. Most (if not all) of the problems are non-trivial, but our feeling is that in time many of them will be conquered, and perhaps by someone who works in an area different from the poser.