ACC for minimal log discrepancies and termination of flips

organized by
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Workshop Summary

Our workshop took place at the American Mathematical Institute from May 14 to May 18, 2012. The organizers were Tommaso de Fernex and Christopher Hacon. There were 26 participants whose interests mostly focused on higher dimensional birational geometry.

The main topic of the workshop is rooted in the minimal model program which generalizes the classification of surfaces to higher dimensional varieties. After the recent proof of the existence of flips (due to Birkar-Cascini-Hacon-McKernan), one of the main remaining open problems in the field is the Termination of Flips Conjecture. As shown by Shokurov, termination of flips can be reduced to a question on minimal log discrepancies, an invariant that gives a sophisticated measure of singularities. Minimal log discrepancies are known to improve after each flip, and Shokurov conjectured that these invariants have no accumulation points from below, that is, that they satisfy the ascending chain condition (ACC). This conjecture, together with a conjecture on the semicontinuity of these invariants, is known to imply the termination of flips. Results on minimal log discrepancies are of independent interest as these invariants are important in the study of singularities.

The above conjectures constitute the main topics for the workshop:

- Termination of flips.
- ACC for minimal log discrepancies.

The workshop was organized as follows. Each morning there were two lectures and in the afternoons we ran moderated problem sessions and we split up in to small groups of approximately 5 participants to work on some of the proposed research problems.

Some of the morning lectures were of a more introductory nature and some discussed more recent advances and breakthroughs. Ambro delivered an introductory lecture defining minimal log discrepancies, discussing their properties and introducing the main conjectures related to these invariants. Shokhurov delivered a lecture on his result showing that the ACC for minimal log discrepancies and the conjecture on the semicontinuity of minimal log discrepancies implies the termination of flips. He then discussed further results and conjectures related to the ACC for minimal log discrepancies. Borisov discussed the proof of the ACC for minimal log discrepancies in the case of toric varieties. Prokhorov described recent work in progress which shows that the BAB conjecture implies two questions of Serre concerning the structure of the Cremona group. Mustaţă's lecture was on the topic of m-adic semicontinuity for log canonical thresholds and minimal log discrepancies, and Kawakita discussed his recent exciting partial results towards the ACC for minimal log discrepancies based on the approach proposed by Mustaţă. Schwede gave an overview of invariants and techniques developed in characteristic $p > 0$ which are closely related to log canonical threshold and minimal log discrepancies. In a series of two lectures, Hacon and

McKernan gave a detailed account of their recent joint work with Chenyang Xu which proves the conjecture on the ACC for log canonical thresholds (another sophisticated measure of singularities closely related to minimal log discrepancies). It is expected that this result and the techniques developped will be extremely useful in the study of the ACC conjecture for minimal log discrepancies. Finally, Shokurov delivered a second lecture on the dual complex of the log canonical singularities.

There were a wide variety of problems proposed in the moderated problem session and further discussed in small working groups. These ranged from questions related Kawakita's work and the m-adic semicontinuity of minimal log discrepancies, explicit computations of minimal log discrepancies related to K-stability, bounds on the singularities of the image of a Mori contraction, toric computations, questions about rational connected varieties, varieties of Fano type and the abundance conjecture.

We felt that the workshop was very successful. We feel that we have learnt a great deal about recent advances in the field and the many interesting open problems in this area. We hope that the excitement and momentum will translate in to further breakthroughs in this active area of research in the near future. We would like to thank AIM and the NSF for the financial support and the hospitality.