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The AIM REU: individual projects with a common theme

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The principle behind the American Institute of Mathematics (AIM) REU is that mathematicians with an active research program have ideas for more projects than they have time to work on. By enlisting the help of enthusiastic undergraduates, it is possible to broaden the range of problems in which they are actively involved.

The AIM REU is run by David Farmer and Brian Conrey, along with one or two of their current postdocs. Conrey and Farmer, and their postdocs, have research interests in analytic number theory: the Riemann zeta-function, modular forms, random matrix theory, elliptic curves, etc. The REU projects all come directly from their ongoing research programs, so there are natural relationships between the projects. The fact that there is an underlying theme between the various student projects is one of the strengths of the program. There is a wide variety of interaction among the students and the advisers, with all the students developing an interest in each other's projects, and all the advisers being able to work with all the students. The students become part of an active research community.

1. Key features of the AIM REU

Individual projects. Every student chooses their own individual research problem. This allows the student to take full responsibility for their own project and more closely models traditional mathematics research.

The students all work on related projects, although it often takes them many weeks to understand the connections! Through weekly progress reports and a variety of daily interactions with the advisers and other students, the participants spend a lot of time talking to each other, explaining their work, and learning about each other both on a mathematical and a personal level.

By having all the projects in related areas we are able to foster a sense of community where everyone feels a part of a larger endeavor. This allows us to have all the students work on separate projects without the likelihood of feeling isolated.

Immediate work. Students choose their problem on the first day and begin working immediately, without any preliminary background reading. This feature is somewhat unusual, considering that the problems are in an area that is considered to require a reasonable amount of background. We elaborate on this in the next two sections of this paper.

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Ongoing research. The student projects arise naturally from the research programs of the mathematicians running the REU. One consequence is that the specific projects cannot be finalized until the last minute. While it is possible to give a general idea of the program (students usually want to know the possible projects and how their particular project will be decided), we always have to consider recent developments before the beginning of the REU.

Postdoctoral training. The postdocs are involved in all aspects of the REU: everything from selecting appropriate projects to helping the students prepare their talks in the final week. This is valuable training because there is an increasing expectation for junior faculty to advise undergraduate projects (especially at liberal arts colleges).

This "training" begins shortly before the program when the list of possible projects is developed. It is not required that the postdocs contribute a problem (although they usually do). The senior mathematicians and postdocs have 2 or 3 meetings which start with a discussion of ideas for projects. The senior mathematicians examine all aspects of the proposed problems, being as explicit as possible about which properties make a problem suitable, or not, for the REU. By the final meeting, the discussions turns to the critical issue of how to introduce the problems to the student so that they can make a reasonable choice for their project and begin working immediately. This process is not easy to capture on paper, so it is a big help for the postdocs to see it in action.

Regular meetings between the directors and the postdocs occur during the program. These are used to monitor student progress and also to discuss issues such as: what is the right amount of help to give a student who is "stuck," how to prepare a student to give a talk, etc. These meetings are in addition to the daily interaction of the mentors and the students, which gives the postdocs an opportunity to observe the more experienced mentors and to call on help as needed.

Mathematica. Many of our REU projects begin with experimentation in *Mathematica*, and in many cases the work is done primarily with the help of *Mathematica* and other computer-algebra systems.

Standard topics. The AIM REU has many facets which feature in most REUs, but we mention them for completeness. Throughout the program there are lectures on material related to the student projects. All the students learn LaTeX and write a final report in the style of a research paper. Also, the students give a formal talk on their work and are encouraged to give a talk when they return to their home institution. The mentors and the students all see each other several times every day and often eat lunch together. Each student has their own desk in a shared office.

2. Identifying appropriate projects

Prior to the REU we must choose projects which allow our students to select a problem on the first day and begin work immediately, after only a few verbal preliminaries and without any background reading. We have identified three key criteria for deciding if a problem is appropriate for this approach, and one "non-criterion" which is best ignored.

1. How would I start to work on the problem? The problem is part of my research, so obviously I find it interesting. Suppose I had time to work on the problem: *what would be my first step?* If I don't have an answer to that question, then clearly the problem is not appropriate for a student. On the other hand, if I have a good idea of what I would do, then it is possible that the problem may be good for the REU.

2. Could an undergraduate execute my first steps on the problem? I know what I would try if I were to work on the problem. Would an undergraduate be able to do that work? For example, if I would start with some numerical experiments in *Mathematica*, or I would start by considering the problem for some low-degree polynomials, or if I would enumerate the first few cases by brute-force, then perhaps a student would be able to do it, too.

Note: this question only asks if the student could do the work, at this point we don't consider whether or not the student could understand *why* that work relates to the problem.

3. It there something related to the problem which the student could begin doing immediately? An effective way to engage a student on a problem is to have them begin work on the first day: work which will lead them to see something interesting. This only needs to be in the same general area as the intended problem, because its purpose is to quickly put the student into a research-like environment and to start them towards an understanding of the problem. Once they have seen something new (to them), they want to understand their observations (which motivates them to learn background material), and they will want to keep going (which motivates them to get to the real problem).

For example, I have had several students do research on modular forms. A good first task for them is: use *Mathematica* to expand the following infinite product into a sum:

$$q\prod_{j=1}^{\infty}(1-q^j)^{24} = \sum_{n=1}^{\infty}\tau(n)q^n. \tag{1}$$

It doesn't take much knowledge of *Mathematica* to truncate the product on the left and expand it into a sum, which gives the first several values of $\tau(n)$. The student will "discover" that $\tau(2)\tau(3) = \tau(6)$ and $\tau(2)\tau(5) = \tau(10)$, and in general $\tau(n)\tau(m) = \tau(nm)$ if n and m are relatively prime. The proof of this result is accessible to an undergraduate (although it is not found in the undergraduate curriculum), and the student is motivated to see the proof.

4. You don't need to fill in the gap. After the student has started, you still have to chart a course to their chosen problem. While it is helpful to have an idea of this before the REU begins, I have not found this to be critical. You will work together with the student to find a path from the initial task to the problem. You have already determined that both ends of that path are accessible to the student, so you have to trust that you can connect them in an understandable way. Often you can identify a huge chasm which seems hopeless for the student to breach. In these cases a few carefully chosen "black boxes" (which the student will have to accept on faith) can be helpful. Your goal is not to give an entire graduate course in the subject, but to put them in a position to work on their problem.

It takes a lot of work to bring a student up to speed on a problem, and this is best done in partnership with them because you need to take into account their particular strengths and weaknesses.

3. The beginning of the project

Ten weeks (or less) is a very short amount of time to accomplish meaningful research, so it is important for students to begin right away. In the AIM program a variety of possible projects are described on the first morning. Shortly after lunch each student chooses a problem and begins work. Students are never asked to read or learn anything before beginning their project, and the students are not formally introduced to any of the background of their project until their work is well underway.

Real work begins the first day. This is the aspect of the program which many people find surprising, including the students. The first morning all the mentors describe possible projects. Technical terms are suppressed as much as possible, and the emphasis is on the *methods* that the project will involve. For example, some projects involve a lot of computer work, others have a more geometric flavor, etc. By lunchtime all the projects have been described, and the students are told that after lunch they can ask as many questions as they want, after which they must choose a project and begin working immediately. They are not forced to stay with that project if they don't like it, but they have to give it a serious try. Should two people want to do the same project, the mentors can usually find two independent sub-projects with the same theme. The mentors and students eat lunch together and then return to AIM.

The after-lunch meeting usually takes an hour, and after the student questions are answered we leave them alone for some time to talk it over. Some students are uncomfortable with this process, and many explain that they would like to learn more about the problems before choosing. We tell them: "You have spent years learning mathematics, and you will have plenty of time to learn more mathematics during the semester. If you spend too much time learning the background, then you won't have time to *do* the mathematics."

Starting a project. How are they are supposed to start working without any background? The answer is to give them specific tasks which they can do using only what they already know, and which bear some relation to their project. The tasks should lead them to see some interesting phenomenon. This could involve a pen-and-paper calculation, or a computer calculation (we use *Mathematica* a lot), or drawing a picture, etc. There is no need to explain the connection between the task and the project – that will come later.

The students complete their first task, which hopefully leads them to an interesting observation. Now they are hooked. They want to move on to what is next, they want to know the relationship to their project, they want to understand the background, etc. Of course the student will have to learn some background at some point, but that can be done in parallel with the "real" work, and the research provides the motivation. You have asked them to trust you and begin without knowing where they are going, but once they get started you have to provide them the means to fill in the gaps.

I intend this approach to be an explicit rejection of the following method, which has no place in an REU: "Go read these books and papers and when you understand them come back and I'll tell you about your project."

I do not claim that every research project can be started in this way. But a very large number of those *which are suitable for REU projects* can indeed fit our model. It takes some time and creativity, and this is a key part of the process of identifying suitable projects.

Students have individual projects. Each student has his or her own separate project. This project is under the primary supervision of one of the mathematicians, although at some point during the summer every mathematician works with every student. This helps expose the student to different perspectives and also makes it possible for the mathematicians to go to conferences without compromising the attention given to the students.

4. Miscellaneous

Some features of the AIM REU which we recommend for other programs:

Conference call prior to the summer. I arrange a conference call with all the students who have an offer to join our REU. This is efficient because there are many questions which nearly all students have, so I can answer them all at once. It also helps the students to find out a little about each other and to begin building a sense of community.

Weekly reports. Every Monday morning all the students give progress reports. We specify that the reports should be self-contained and should not assume the audience recalls the details of their previous report. What is their project and why is it interesting? What did they do last week? What is the overall plan and what specifically will they do this week?

We meet with the students before the report to discuss what they will say. Usually they need encouragement to be expansive about the "big picture." We meet with them again after the report to offer constructive criticism, usually phrased in the form: "Next week you might try..."

Initially the students do not like giving these reports, but in our exit surveys they tell us that the reports were valuable and they are glad they did them. In addition to making them more comfortable about giving a formal presentation of mathematics, they specifically indicate that the reports help them to keep their project in perspective and to plan their week.

LaTeX at the first opportunity. Have the students learn LaTeX the first time they do any work that could be part of their final report. This could be original work, or it could be some standard calculation that they worked through which could be part of the introduction in their paper. Give them a simple template file, and just have them type something small. Don't ask for a formal writeup or introduction: just have them start learning how to LaTeX. Encourage them to keep writing up anything that could be useful.

Preparing final talks. We host a mini-symposium during the final two days of the program where all the students give 40-50 minute talks. We treat this just like a session at a conference.

Sally Koutsoliotas and I have developed an effective approach to preparing students to give talks. We prepared a guide, available at `aimath.org/mathcommunity/`. Here is a summary: it is based on a series of four meetings.

The first meeting is just a conversation with the students. We identify the main point of their talk and work backward to determine the absolute minimum amount of information needed to *understand* and *appreciate* their main point. The student goes away and prepares a rough outline of their talk, usually in the form of handwritten drafts of slides on paper. (It is critical that the student prepare something very rough, because it will probably have to be completely taken apart and you don't want them to resist your suggested changes).

The second meeting begins with the student summarizing their main point, *and then* presenting their rough draft by laying it all out on a table. The mentor and student work together to identify the different parts of the talk, stressing the importance of the *transition* between each part, and the *flow* of ideas. The student goes away to make a complete version of their slides.

The third meeting involves laying out the slides on a big table (use printouts if the talk is on the computer) and grouping them according to the different parts of the talk. We evaluate the total number of slides, the content and layout of each slide, and the all-important transition between the different parts of the talk. The student goes away and makes final versions of the slides.

The fourth meeting is the first time the student actually gives a practice talk. Tell them to just talk and don't worry about how long it will take (you don't want them rushing at the end: if material has to be cut, it probably isn't at the end of the talk). Then sit down together at a table with all the slides laid out, and try to discuss only the 4 or 5 most important things they need to change. If appropriate, that is, if they have to make extensive revisions, schedule another practice talk. If there are minor changes, such as the talk was slightly too long, you may want to suggest that they practice in front of a friend.

Sample papers. To help the students in their final writeup we prepare a folder of sample papers. These are research papers from the general subject area of the REU and which have good overall organization, a well-written abstract, and an accessible introduction. These papers are models to show the student what their final paper should look like.

5. The benefits, mathematical and otherwise

Running an REU is a rewarding experience, particularly when you can quickly bring students up to speed and make them part of your research group. However, it is a huge amount of work, and to do it right you have to be willing to dedicate the majority of your time to it.

Is the total mathematical output larger than than it would have been if, instead of the REU, I just spent all day working on my own research? I think the answer is 'yes,' but in some sense the question is meaningless because I would not have the stamina to focus all-day-every-day on my research. The REU definitely increases the breadth of projects I am involved in, and that makes me a better mathematician.

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