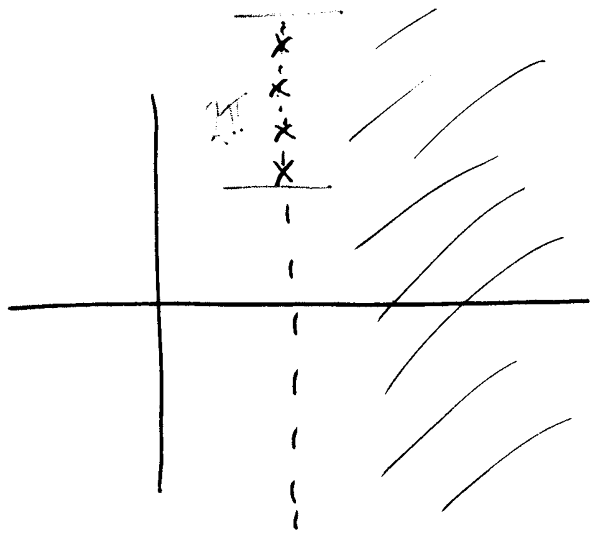
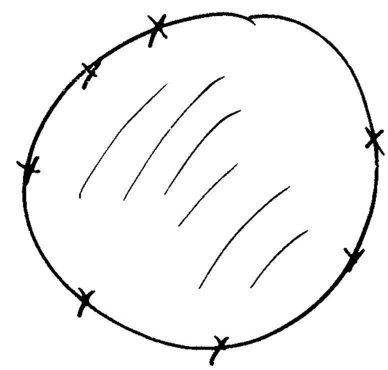
Why set $N = \log \frac{T}{2\pi}$?

Catch-up #1

Farmer





$$s \longrightarrow e^{\frac{1}{2}-s}$$

To model moments: $\quad L(\tfrac{1}{2}+it+\alpha) \longleftrightarrow \Lambda(e^{-\alpha})$

$$\left\langle \begin{array}{l}\text{\# of zeros}\\ \text{in a block}\end{array} \right\rangle = N(T+2\pi) - N(T)$$

$$\approx 2\pi N'(T)$$

$$= 2\pi\left(\frac{1}{2\pi} \log\left(\frac{T}{2\pi e}\right) - \frac{1}{2\pi} T \frac{1}{T} \right)$$

$$= \log \frac{T}{2\pi} \qquad \text{error } N''(T) = \frac{1}{T}.$$

$\|$

N

$\vee$

Matrix size

Equivalently, equate the density of zeros

In general, set

matrix size = log conductor

$$L(s) = \varepsilon \, X(s) \, \bar{L}(1-s)$$

$$\xi_L(s) = Q^s \prod_{j=1}^{J} \Gamma(\lambda_j s + \mu_j) \, L(s) = \varepsilon \, \bar{\xi}_L(1-s)$$

The log conductor of L is

$$c(L) := \left| \frac{X'}{X} \right| = |X'|$$

Recall: $|X(\tfrac{1}{2}+it)| = 1$

So $\dfrac{2\pi}{c(L)}$ = local density of zeros (use F.E. and argument principle)

<u>Example</u>: $L(s, \chi_d)$: $\left(\frac{d}{\pi}\right)^{s/2} \Gamma\left(\frac{s}{2} + \kappa\right)$

Symplectic family

$$2N = \log\left(\frac{d}{\pi}\right) + \frac{\Gamma'}{\Gamma}\left(\frac{1}{4} + \kappa\right)$$

<u>Exercise</u>: Verify that this definition gives $N = \log \frac{T}{2\pi}$ for zeta.

Note: There are other less precise notions of conductor.