Practical and Theoretical Aspects of Time-Reversal Communications

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Overview

- **TR real-world demonstration:** Design of a testbed for outdoor TR-transmission (with D. Baum, A. Paulraj)
- **Focusing capabilities of TR:** System performance evaluation for a wireless communications perspective (with M. Emami, Th. Strohmer, M. Vu, A. Paulraj, G. Papanicolaou)
- **MU-TR:** Notes on achievable rates (with A. Kapur)
- **Radio Channel Characterization:** TR-WSSUS channels (with H. Bölcskei)

Requirements for a Wireless TR Testbed

in order to demonstrate focusing, we need

- **multiple antennas**
- **high delay spread bandwidth** product ($\sigma_\tau W > 20$), i.e.,
  - large **bandwidth**
  - large **distance** between transmitter and receiver
  - large **transmit power**
  - large **angular spread**
  - low-gain **antennas**

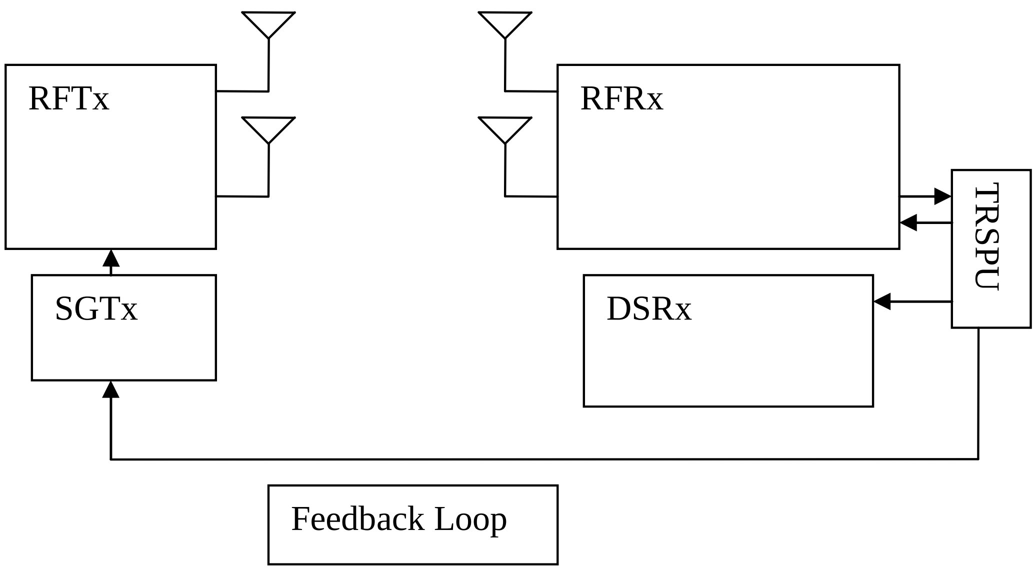
Convert Commercially Available Channel Sounder into TR Testbed

use **wide band, correlation based MIMO channel sounder**

for **TR demonstration** we need to apply the following changes:

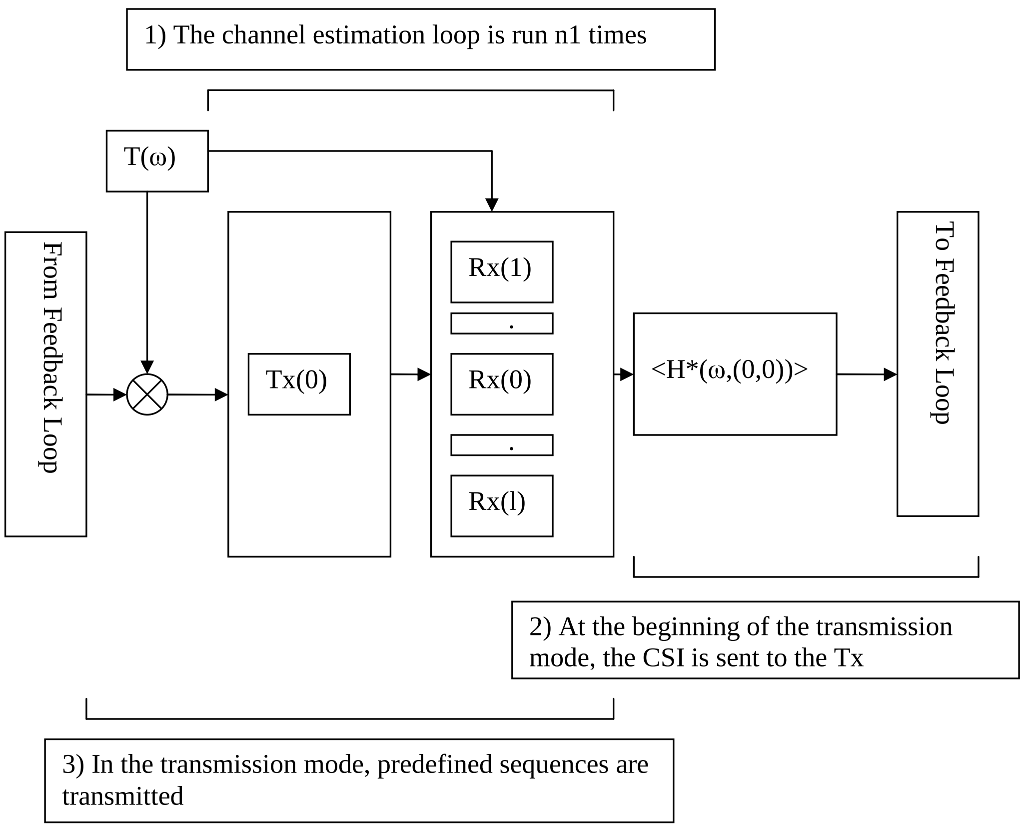
- feed back estimated channel state information to the transmitter
- convolve code sequence at transmitter with this CIR
- use several receive antennas (SIMO) to demonstrate spatial focusing
- use several transmit antennas (MISO) to enhance the focusing effect of TR

Sketch of Modified MIMO Channel Sounder



- a feedback loop connects transmitter and receiver
- a time-reversal signal processing unit (TRSPU) coordinates feedback of CSI and channel estimation at the receiver

Coordination of Channel Feedback and Channel Estimation



- **3 step** procedure
  1. channel estimation (SISO)
  2. channel information feedback
  3. time reversal transmission (here: switched SIMO)

The Feedback Loop — Required Data Rate

the feedback loop is the **bottleneck** of the sounder

- assume CIR has 1000 taps
- complex symbols, 8 bit for real/imag part
- static channel with Doppler of 1 Hz
- update once per 0.2s
- required data rate is 80kbps **guaranteed and without delay**

The Feedback Loop — Potential Candidates

several options for the feedback loop have been analyzed

1. **Cellular Systems**
   - ubiquitous
   - data rate $\mathcal{O}$(100kbps), but unreliable delays (link-built up)

2. **Microwave Link**
   - heavy to handle
   - data rate $\mathcal{O}$(Mbps), reliable communication once link is established

The Feedback Loop — Potential Candidates

3. **Ethernet / WLAN**

   - available only in urban areas
   - data rate $\mathcal{O}$(Mbps), potential delays through traffic

4. **Optical Fibre**

   - restriction in length
   - very high data rate, no delay

for the present design, we opted for solution 3. and 4.

Antennas and Link Budget

compute link budget against bandwidth and antenna gains for
30 dBm transmit power, 33 dB correlation gain, 20 dB target SNR

	q=3	q=4
120 MHz	4300 m	530 m
240 MHz	3400 m	450 m

pathloss and bandwidth against range at gains of 6 / 10 dBi

	q=3	q=4
3^* / 3^* dBi	2000 m	300 m
6 / 6 dBi	2500 m	350 m

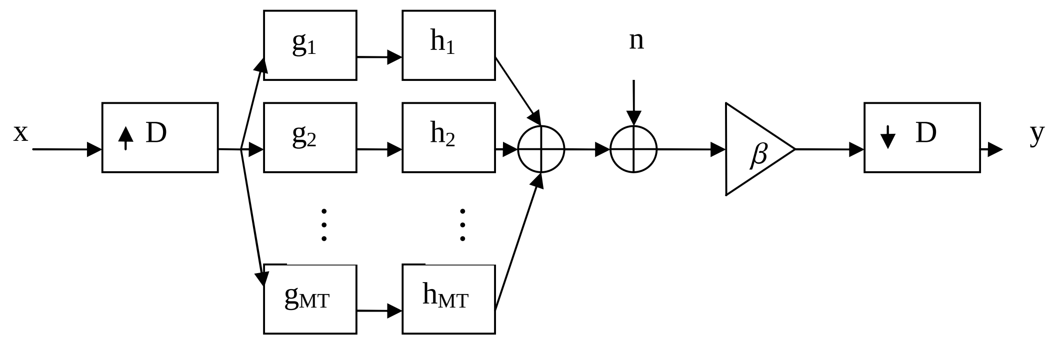
pathloss and antenna gains against range at 240 MHz (*: higher P_{TX})

delay spread bandwidth product should be $> 20 \quad \Rightarrow \sigma_\tau > \mathcal{O}(100ns)$

Summary of TR Demonstration

- the design of a TR sounder has **demanding requirements**
- a **commercially available** MIMO channel sounder can be converted into a TR sounder
- with this sounder, **temporal and spatial focusing** can be demonstrated
- demonstration will hopefully take place in spring 2005
- more sophisticated **transmit schemes** are desirable in future

Temporal and Spatial Focusing in a Simple TR System



- consider a simple communication system in a wideband channel
- the system has M_T transmit and a single receive antenna
- use linear precoder / TR precoder
- single tap at receiver

Signal Model

- input-output relation of channel is

$$y[k] = \beta \mathbf{x}_k \mathbf{H} \mathbf{g} + \beta n[k]$$

$\mathbf{x}_k = [x_1 x_2 \dots x_{2L-1}]$: vector of input symbols,
$y[k]$: output symbol, $n[k]$ noise,
$\mathbf{H}$: block toeplitz channel matrix,
$\mathbf{g} = [\mathbf{g}[0]^T \quad \dots \quad \mathbf{g}[L-1]^T]^T$,
$\mathbf{g}[l] = [g_1[l] \quad \dots \quad g_{M_T}[l]]^T \quad l = 0, \dots, L-1.$

Optimal Linear Precoder & TR Precoder

- optimal $\mathbf{g}$ and β for no rate-back off:

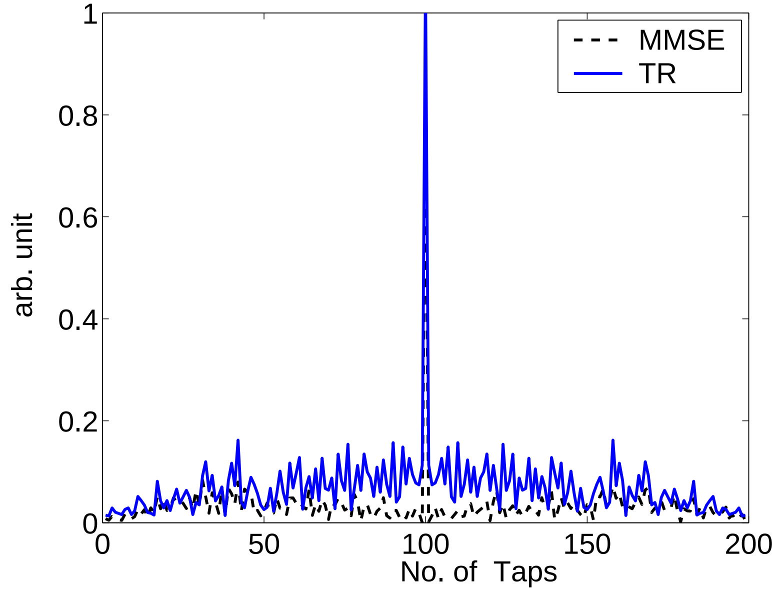
$$\hat{\mathbf{g}} = \frac{1}{\beta}\left(\mathbf{H}^H\mathbf{H} + \frac{\sigma^2}{P_{T_x}}\mathbf{I}\right)^{-1}\mathbf{H}^H\hat{\mathbf{e}}_{\boldsymbol{\Delta}}$$

$$\beta^2 = \hat{\mathbf{e}}_{\boldsymbol{\Delta}}^T\mathbf{H}\left(\mathbf{H}^H\mathbf{H} + \frac{\sigma^2}{P_{T_x}}\mathbf{I}\right)^{-2}\mathbf{H}^H\hat{\mathbf{e}}_{\boldsymbol{\Delta}}.$$

- TR precoder is obtained at low SNR

Δ: delay of the equalizer,
$\hat{\mathbf{e}}_{\boldsymbol{\Delta}}$: unit vector with 1 in entry Δ
$P_{T_x} = E[\mathbf{x}_k^T\mathbf{x}_k]$, σ^2: noise power.

Focusing Capability



- focusing capability of the optimal precoder is higher
- optimal precoder needs matrix inversion

System Improvement by Rate Back-off

- ISI and CCI can be reduced by rate back-off by a factor of D
- signal model is now ($D = 1, \ldots, L$)

$$y^{[D]}[k] = \beta \sum_{i=1}^{M_T} \sum_{l} (h_i \star g_i)[Dl]x[k-l] + \beta n[k]$$

$y^{[D]}[k]$: output symbol received with rate back-off.

Performance Measures

- define focusing ratio κ

$$\kappa = \frac{E[P_0]}{E[Q^{[D]}]}$$

$E[P_0]$: expected power in the central peak,
$E[Q^{[D]}]$: ISI power with rate back-off D

- P_0 and $Q^{[D]}$ are

$$\begin{aligned} P &= \left| \sum_{i=1}^{M_T} \sum_{l=0}^{L-1} h_i[l] g_i[-l] \right|^2 \\ Q^{[D]} &= \left| \sum_{i=1}^{M_T} \sum_{l \neq 0} \sum_{k \neq 0} h_i[l] g_i[m] \delta[Dk - m - l] \right|^2 . \end{aligned}$$

The Effective SNR

- define effective SNR ρ_{eff}

$$\rho_{eff,D} = \frac{P_{Tx}E[P_0]}{P_{Tx}E[Q^{[D]}] + \sigma^2} = \frac{E[P_0]}{E[Q^{[D]}] + \frac{1}{\rho}},$$

$\rho = \frac{P_{Tx}}{\sigma^2}$.

- define ρ_{MFB}

$$\rho_{MFB} = \frac{P_{Tx}E[\sum_{i=1}^{M_T} \sum_l |h_i[l]|^2]}{\sigma^2}.$$

Application to TR Systems

- compute κ for TR filter in channel with exponential PDP and delay spread σ_τ

$$\kappa_{TR} = \left(1 - \exp - \frac{D}{\sigma_\tau}\right) \times \frac{\left[M_T^2(1 + \exp - \frac{1}{\sigma_\tau}) + M_T(1 - \exp - \frac{1}{\sigma_\tau})\right]}{2M_T \exp - \frac{D}{\sigma_\tau}\left(1 - \exp - \frac{1}{\sigma_\tau}\right)}.$$

- effective SNR with rate back-off is

$$\rho_{eff,D} = \rho_{MFB} \frac{1}{1 + \frac{\rho_{MFB}}{\kappa_{TR}}}.$$

Performance Evaluation of TR Systems

- limiting cases:

$$\rho_{eff,D} = \rho_{MFB} \quad \text{for } \sigma_\tau \longrightarrow 0$$

$$\rho_{eff,D} = \frac{\rho_{MFB} D M_T}{\rho_{MFB} + D M_T} \quad \text{for } \sigma_\tau \longrightarrow \infty$$

- evaluation of the latter gives

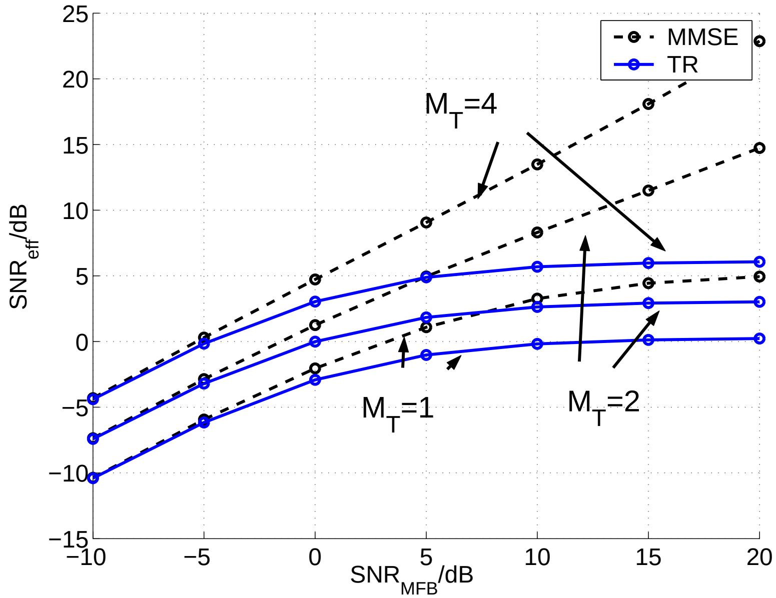
$$\rho_{eff,D} = \rho_{MFB} \quad \text{for } \rho_{MFB} \text{ low, } \sigma_\tau \text{ large}$$

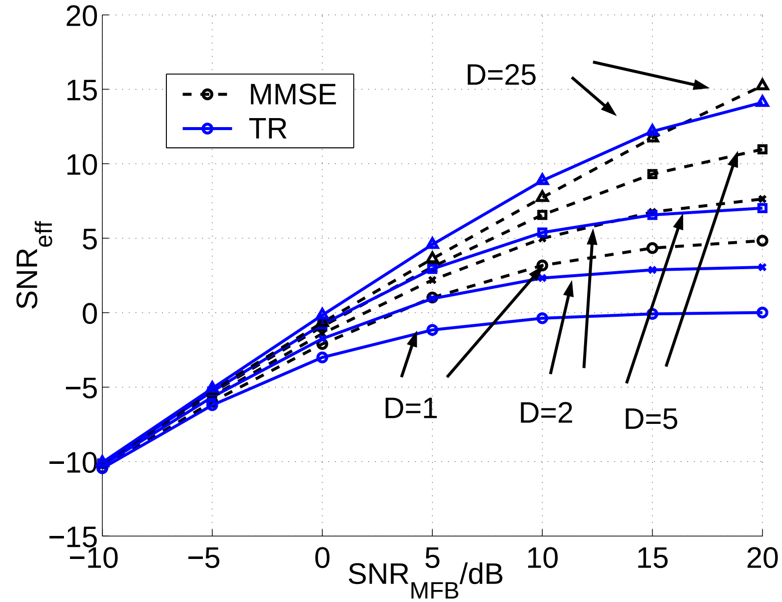
$$\rho_{eff,D} = D M_T \quad \text{for } \rho_{MFB},\ \sigma_\tau \text{ large.}$$

Interpretation of the Results

- TR is lossless if delay spread of channel is very small
- TR is optimal at low SNR
- performance of a TR system at high input SNR saturates at a value independent from the delay spread of the channel

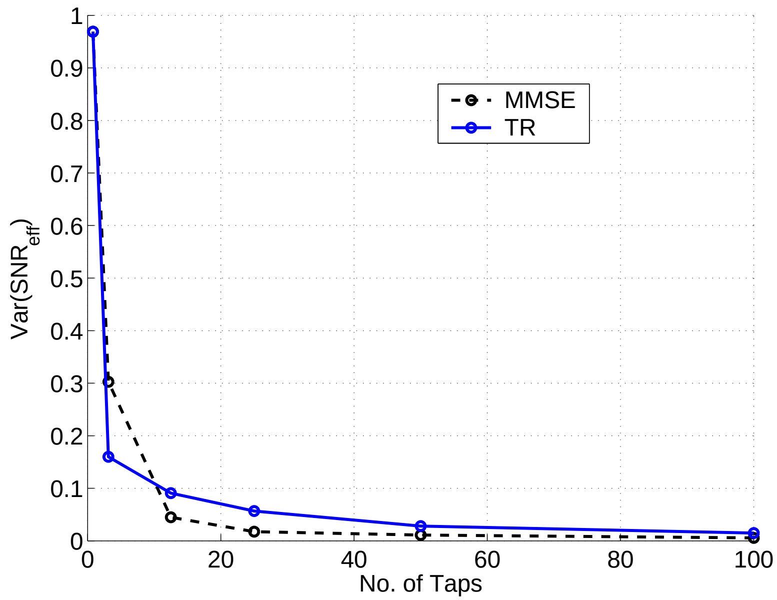
Plot of Analytical and Simulation Results





- 3 dB SNR increase per additional transmit antenna
- 3 dB SNR increase per doubled D

Channel Hardening

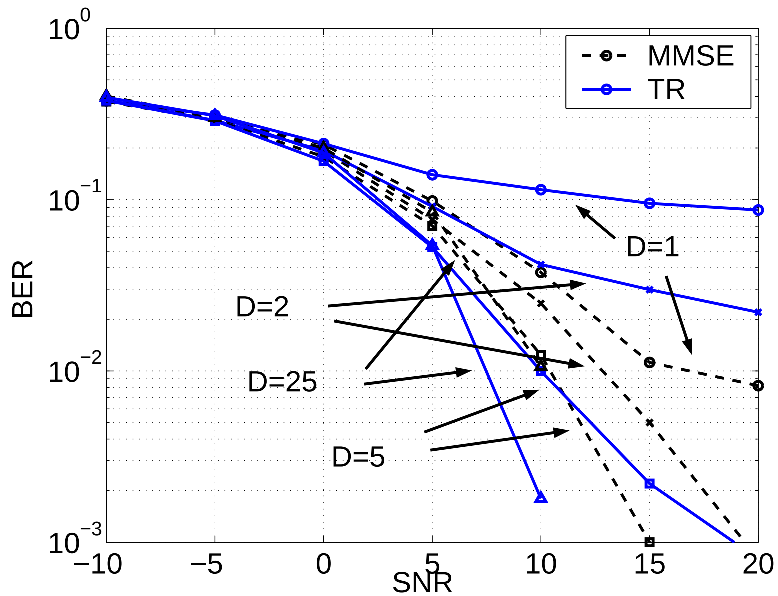


compute channel hardening

$$\sigma^2_{SNR} = \text{Var}\frac{P_{Tx}P_0}{P_{Tx}Q^{[D]} + \sigma^2} / E[\frac{P_{Tx}P_0}{P_{Tx}Q^{[D]} + \sigma^2}]^2.$$

for $\rho = 10$ dB and $M_T = 1$

Bit-Error-Rates of the System



compute BERs for $D = 1, 2, 5$, and 25 and $L = 100$ taps.

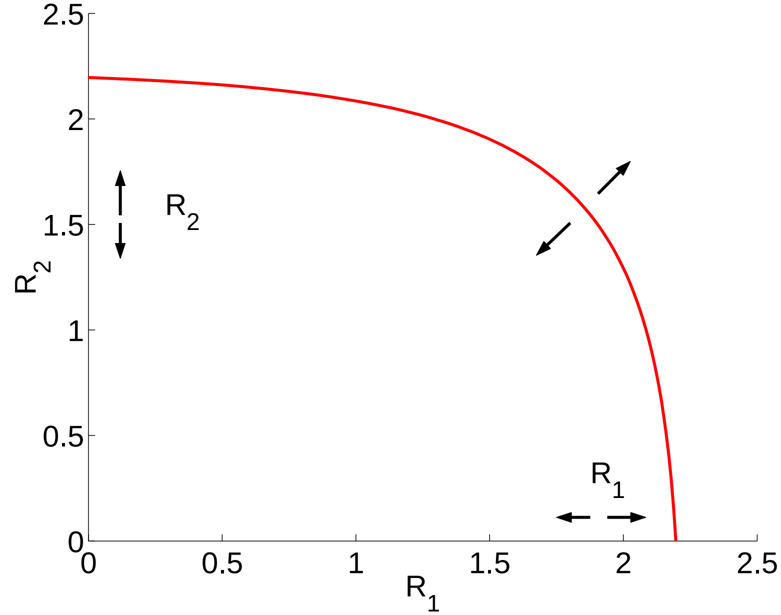
Conclusion

- the computation of interference power for TR systems in wireless communications requires to take the power in the sidelobes into account
- at high rates, this power can be significant
- TR is powerful at low SNR
- TR is advantagous for systems with simple receivers when bandwidth can be traded against ISI

TR in a Broadcast Channel

- in an SU system, TR has the advantage of good outage properties
- it trades increased bandwidth against better outage capacity by using a very simple power allocation scheme
- in a BC channel, TR will suffer from spatial interference
- goal is to characterize outage capacity regions for the BC and the MU-TR channel
- (assume that this is a problem from a more formidable class)

Goal



- describe the hardening of the rate region due to the hardening of the individual users' rates

- find intelligent coding and signal processing schemes that improve the SINR of a MU-TR system

Freq Selective BC Channel

(Goldsmith and Effros, Trans IT, 2001)

- use waterfilling over frequencies and superposition coding

$$
\begin{aligned}
R_1 &\leq \frac{1}{2\pi}\int_{-\pi}^{\pi} C\left(\frac{\alpha(\omega)P(\omega)}{N_1(\omega)}\right) 1[N_1(\omega) \leq N_2(\omega)]\mathrm{d}\omega \\
&+\frac{1}{2\pi}\int_{-\pi}^{\pi} C\left(\frac{\bar{\alpha}(\omega)P(\omega)}{N_1(\omega)+\alpha(\omega)P(\omega)}\right) 1[N_2(\omega) < N_1(\omega)]\mathrm{d}\omega \\
R_2 &\leq \frac{1}{2\pi}\int_{-\pi}^{\pi} C\left(\frac{\alpha(\omega)P(\omega)}{N_2(\omega)}\right) 1[N_2(\omega) \leq N_1(\omega)]\mathrm{d}\omega \\
&+\frac{1}{2\pi}\int_{-\pi}^{\pi} C\left(\frac{\bar{\alpha}(\omega)P(\omega)}{N_2(\omega)+\alpha(\omega)P(\omega)}\right) 1[N_1(\omega) \leq N_2(\omega)]\mathrm{d}\omega
\end{aligned}
$$

with $\bar{\alpha} = 1 - \alpha$ and $\frac{1}{2\pi}\int_{-\pi}^{\pi} P(\omega)\mathrm{d}\omega = 1$.

Simple TR without Superposition Coding

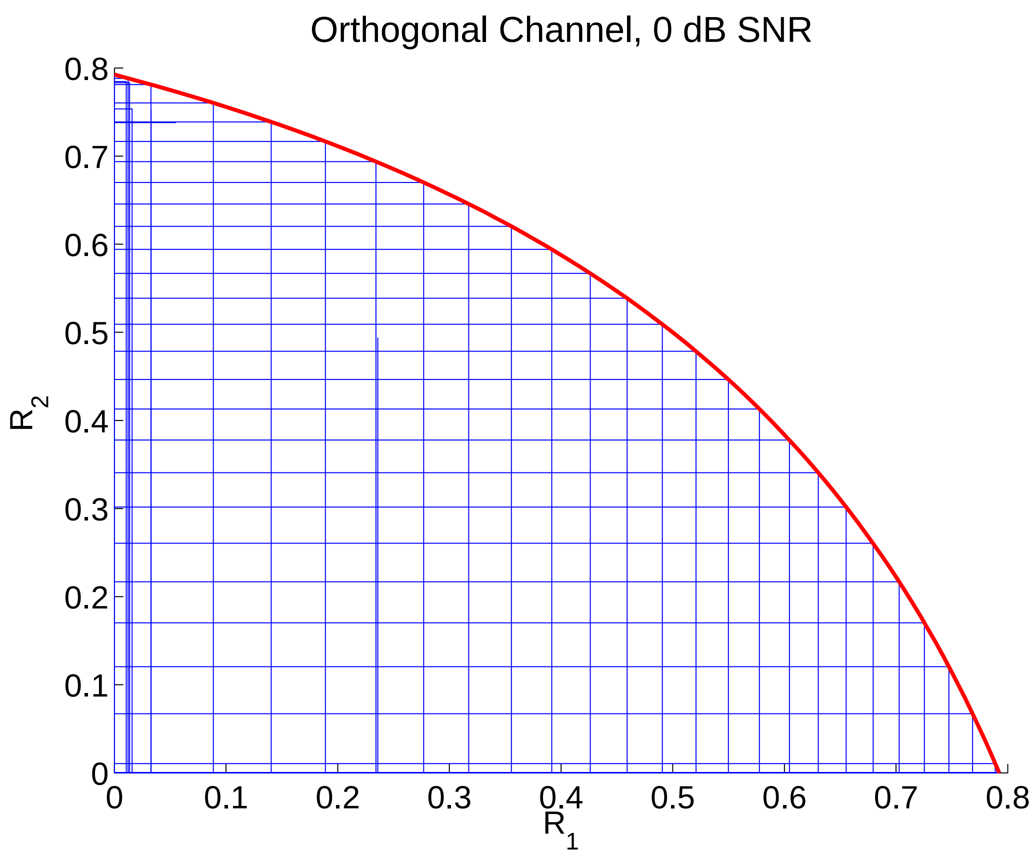
- TR: α, $\bar{\alpha}$ constant over ω;
- noise variation is expressed by fading channel, $N_i(\omega) = \frac{1}{|H_i(\omega)|^2}$
- no superposition coding; $P_i(\omega) = |H_i(\omega)|^2$. For R_1 we have

$$R_1 \quad \leq \frac{1}{2\pi} \int_{-\pi}^{\pi} \log \left(1 + \frac{\alpha |H_1(\omega)|^4}{1 + \bar{\alpha} |H_1(\omega)|^2 \, |H_2(\omega)|^2} \right) \mathrm{d}\omega$$

- without further measures, an interference penalty is likely
- in efficiency, such a system is equivalent to SU-TR with a single-tap receiver
- at high SNR we expect to touch the (1,1)-point of the rate region

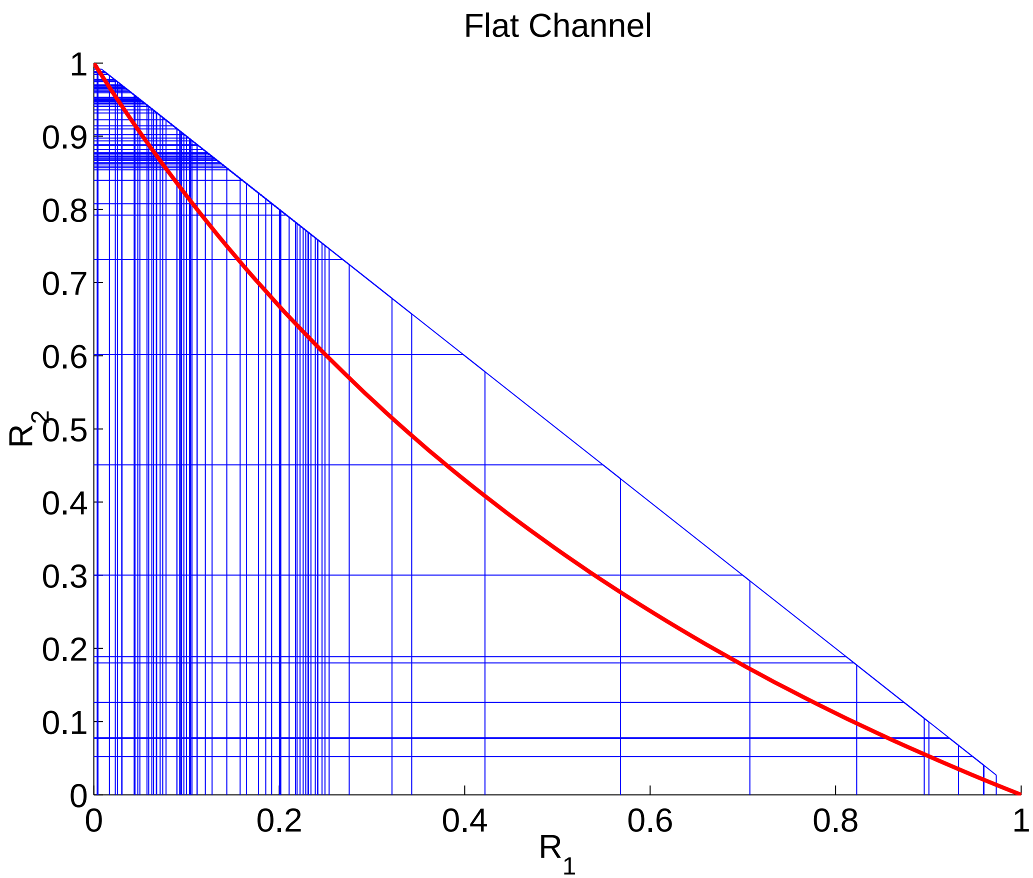
Orthogonal Channels

(thanks to Eric Stauffer)



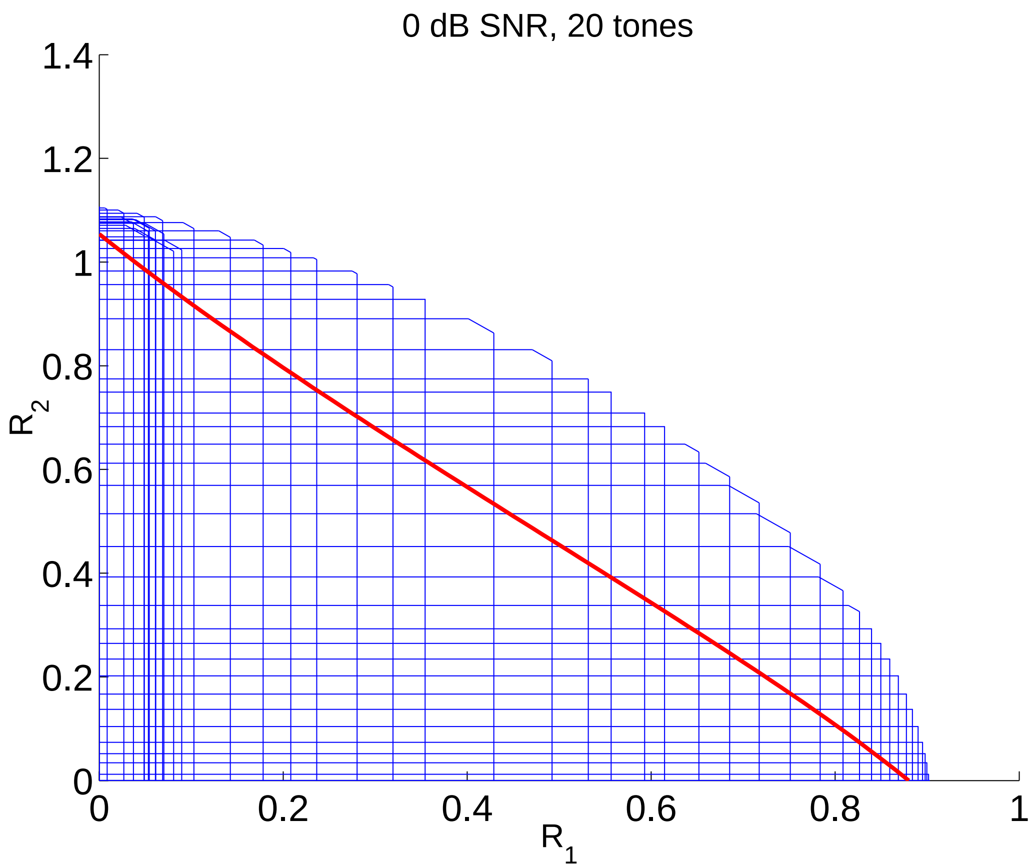
- in an orthogonal channel, TR works perfectly

Flat Channels



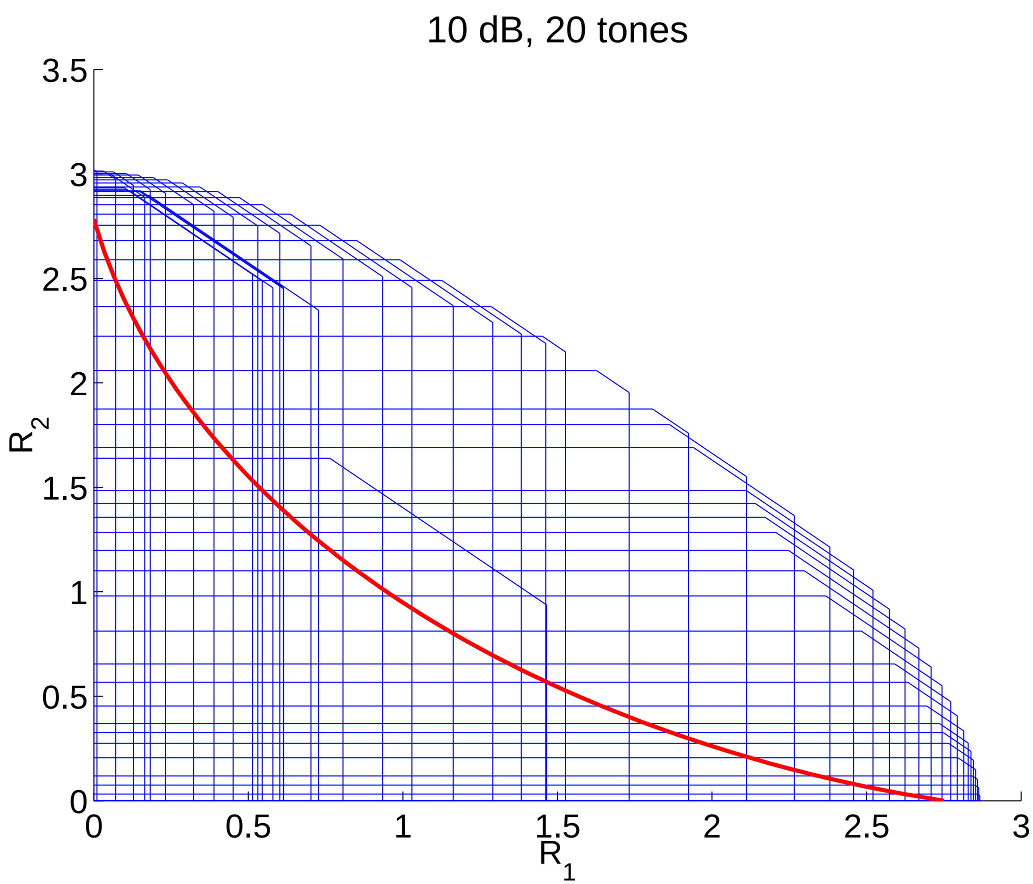
- in a flat channel, TR suffers from interference
- even perfect precoding does not really great

IID Gaussian Channels



- at low SNR, the BC is not much better than TR
- in TR, the frequency allocation is random

IID Gaussian Channels



- at high SNR, the system performance saturates compared to the best imginable system
- the rate region of TR touches the (1,1)-point

Conclusions — Ergodic Capacity Region

- the performance of MU-TR is degraded by interference from other users
- in order to design efficent TR-schemes, we can consider
  - some form of rate back-off
  - some form of scheduling in frequency / in time domain

A Thought Experiment on Spatial Focusing

- from the SU system we know that TR needs excess bandwidth
- we can also define a spatial focusing ratio

$$\kappa_{TR,s} = \frac{\text{power in the peak of the intentionally received CIR}}{\text{powers in the non-intentionally received CIRs at the peak}}$$

- this assumes a single-tap receiver with full rate back-off
- we can conclude that for N interferers,

$$\kappa_{TR,s} \approx \frac{L}{N}$$

A Thought Experiment on Spatial Focusing

- appropriate rate back-off / scheduling is needed due to the only quasi-orthogonality of the code sequences
- what is a good trade-off between the optimal (in the BC sense) and the very simple system ?

Channel Hardening for the MAC and the BC

(with A. Kapur)

- in fact, we want to characterize the cdf of each point of the boundary of the rate region.
- assume Rayleigh channel with iid taps
- start with the simplest case:
  compute rate distributions for the MAC without channel state information
- use suboptimal power allocation scheme over frequency in order to approximate the rate region of the MAC with channel knowledge
- use duality in order to characterize the rate region of the BC channel

MAC Rate Region for no CSIT

- sum rate for fixed number of frequencies

$$C_{MAC,sum} = \frac{1}{N} \sum_{\omega_n} \log \left(1 + \sum_i \rho_i |H_i(\omega_n)|^2 \right).$$

- individual users' rate

$$R_i = \frac{1}{N} \sum_{\omega_n} \log \left(1 + \frac{\rho_i |H_i(\omega_n)|^2}{1 + \sum_{j>i} \rho_j |H_j(\omega_n)|^2} \right).$$

- we expect the channel to harden with N and as a function of the pdf of the term within the sum

MAC Rate Region for no CSIT

- in general, we are interested in the distribution of the random variable I with
$$i = \log(1 + z)$$
where Z is the SINR

- once we know Z, we know a lot about I

- for Rayleigh channels, the pdfs for the SINR have an intriguing form

SINR Distribution for the Sum-rate

- in a Rayleigh channel, we have $z = \sum_{\text{users}} \rho_i |H_i|^2$ and

$$p_z(z) = \sum_i \frac{\gamma_i}{\rho_i} \exp -\frac{z}{\rho_i}$$

with $\gamma_i = \left(\prod_{i \neq j} \left(1 - \frac{\rho_j}{\rho_i}\right)\right)^{-1}$.

- the moments of the rates per tone are

$$E^r(I) = \sum_i \gamma_i E^r(\log(1 + z_i)^r)$$

where z_i is exponentially distributed with decay parameter ρ_i

- $E^r(\log(1 + z_i)^r)$ are the moments of the capacity of an SU Rayleigh fading channel

SINR PDFs for Individual Users' Rates

- pdf of the SINR of the rate R_i, $z = \frac{\rho_1 |H_i|^2}{1+\sum_{j>i} \rho_j |H_j|^2}$ is

$$p_{SINR}(z) = \exp -\frac{z}{\rho_i} \sum_{j>i} \gamma_j \frac{\rho_j + (1 + \Delta_j z)}{(1 + \Delta_j z)^2}$$

$\Delta_j = \frac{\rho_j}{\rho_i}$

- for most cases, the mass of the pdf is concentrated at low z

SINR Distributions for TR

- the pdf of the SU-capacity appears to be not nice:

$$C_{SU} = \frac{1}{N} \sum_{\omega_n} \log \left(1 + \tilde{\rho} |H(\omega_n)|^4\right)$$

- the pdf of the individual SINRs for MU-TR have the form

$$\begin{aligned} z_i &= \frac{\tilde{\rho}_i |H_i|^4}{1 + |H_i|^2 \sum_{j>i} \tilde{\rho}_j |H_j|^2} \\ &= \frac{\tilde{\rho}_i |H_i|^2}{\frac{1}{|H_i|^2} + \sum_{j>i} \tilde{\rho}_j |H_j|^2} \end{aligned}$$

- can we compute the pdfs of these quantities ?

Conclusion

- channel hardening is a central advantage of TR
- for a performance evaluation, we need to compare the rate regions of the BC channel and that of MU-TR
- both can be formidable problems which may only be solvable for special cases
- since TR systems are likely to operate at low SNR, analytical solutions could still be tractable
- these solutions could help to understand the outage behavior of TR systems and design improved schemes

TR-WSSUS Channels

(with H. Bölcskei)

- in wireless communications, the notion of WSSUS channels plays a central role
- the WSSUS channel framework is used to optimize signal design and adapt it to the structure of the channel
- it incorporates the variation of the channel both over time and frequency
- a TR-WSSUS framework could tell us how to signal best if a TR system is given to us.

WSSUS and TR-WSSUS Systems

- central building blocks of the WSSUS framework are the second moments of the time variant transfer function $H(t, f)$ and its Fourier transforms $t \longrightarrow \nu$ and $f \longrightarrow \tau$

- the WSSUS assumption implies

$$E\left[H(t, f)H^*(t + \Delta t, f + \Delta f)\right] = R(\Delta t, \Delta f)$$

- for TR, we need to compute the fourth moments of the channel

$$\begin{aligned} & E\left[H(t, f)H^*(t + \Delta_1 t, f + \Delta f_1)H(t + \Delta_2 t, f + \Delta_2 f)H^*(t + \Delta_3 t, f + \Delta_3 \right. \\ = \quad & R_{TR}(\Delta_1 t, \Delta_2 t, \Delta_3 t, \Delta_1 f, \Delta_2 f, \Delta_3 f) \end{aligned}$$

Gaussian Stationary Processes

- if a process z is circularly complex Gaussian, SSS and WSS are identical
- we have (Reed, IRE Trans. Inf. Theor., 1962)

$$E[\bar{z}_1\bar{z}_2z_3z_4] = R(\bar{z}_1z_3)R(\bar{z}_2z_4) + R(\bar{z}_2z_3)R(\bar{z}_1z_4).$$

- the proof requires the characteristic function of the process

TR-WSSUS

- how can this property of a Gaussian process be extended to the process $H(t, f)$?

- we may assume that the marginals are Gaussian, but is there any assumption about the joint distribution ?

Conclusions

- research on TR offers a wealth of problems
- it is an extremely practical method to shift complexity from receivers to the transmitter
- we gave a very specific overview about some selected, and partly not yet solved problems