

# Concluding Remarks and Suggested References

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## Abstract

These are the concluding remarks from the participants of the workshop on Random analytic functions. These sessions were moderated by J. Maurice Rojas. Reference material for newcomers to the field were suggested by various experts.

**Alice Guionnet:** In large deviation theory there are different set of tools as compared to random matrices. In free probability analogues of random polynomials should be investigated.

**Ioanna Dumitriu:** As many people are interested in both fields one tends to use similar methods. So it might not be that useful to pursue the analogy further. Interesting problems include finding equivalent of Tracy-Widom laws in more general setting. Methods known so far are incomplete.

**Balint Virag:** There is a lot of work to be done in even defining problems precisely. Gaussian entire functions with negative correlations should be studied. In determinantal processes, one needs to answer whether there exists a determinantal entire function and understand the connections between level sets of harmonic functions, log derivatives of characteristic polynomials of Ginibre ensemble.

**Mikhail Sodin:** There is no unified theory of point processes. Several interesting instances have been studied. For example point processes with pattern of invariance. It is not very clear if a unified theory exists.

**Scott Sheffield:** Finding central limit theorems for Gaussian free fields. The case of dominoes was studied by Richard Kenyon. Interesting to consider would be higher dimensions. For example 2 and 3 dimensional free fields. Higher-dimensional analogs of SLE may be related to DNA and protein structures.

**Maurice Rojas:** Random Viro diagrams might provide a unified framework for studying these questions.

**Bernard Shiffman:** Point processes in higher dimensions should be studied. Zeros attract in higher dimensions. The case of  $n$  analytic functions in  $n$  variables must be studied.

**Mario Wschebor:** Asymptotics of correlations of real polynomials were studied. This might shed light on some of Shiffman's questions.

**Manjunath Krishnapur:** Random matrix valued generating functions might provide unified framework for the study of both random matrices and random polynomials. Random matrices and random polynomials both appear as special cases for this.

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**Ofer Zeitouni:** Universality in random matrices must be analysed. Some universality in random polynomial theory appears due to rescaling.

**David Farmer:** In random matrices the randomness is encoded in the zeros. Whereas in random polynomials the randomness is encoded in the coefficients. The Jacobian of the change of variables matrix from coefficients to zeros is the Vandermonde. This explains the appearance of Vandermonde in all these problems.

**Mikhail Sodin:** Levy and Littlewood had studied the properties of zero distributions of Taylor series.

**Maurice Rojas:** A-discriminants should be used more in random polynomial theory. Formula for discriminant is very difficult. Instead one should attempt to find probability of being within a certain distance of the discriminant.

This discussion concluded with a call for good references, for graduate students interested in entering this general area of research. Various such sources are listed below.

Large Deviations Techniques and Applications, Amir Dembo and Ofer Zeitouni, Jones and Barlett Publishers, Inc., Boston, 1993.

Matrix kernels for the Gaussian orthogonal and symplectic ensembles ( Noyaux matriciels pour les ensembles gaussiens orthogonaux et symplectiques ), Craig Tracy and Harold Widom, Annales de l'institut Fourier, 55 no. 6 p. 2197-2207 , 2005.

The Kernel Function and Conformal Mapping (Mathematical Survey, No 5), Stefan Bergman, American Mathematical Society, Providence R.I., 1970.

Real analysis, G. B. Folland, Wiley, New York, 1984.

Orthogonal polynomials and random matrices : a Riemann-Hilbert approach, Percy Deift, American Mathematical Society, Providence, R.I., 2000.

Zeros of Gaussian analytic functions, Mikhail Sodin, arXiv.org:math/0410343, 2005

Some Random Series of Functions, Jean-Pierre Kahane, Cambridge University Press Publication, London, 1993.

Large deviations and stochastic calculus for large random matrices, Alice Guionnet, Probability surveys, 2004.

The Geometry of Random Fields, R. Adler, Wiley, London, 1981.

Determinantal Processes and Independence, , J. Ben Hough, Manjunath Krishnapur, Yuval Peres, Balint Virag, arXiv.org:math/0503110, 2005.

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