Discussions at the Random Analytic Functions Workshop at AIM: Part I

Swaminathan Sethuraman*
Texas A&M University
Department of Mathematics
TAMU 3368
College Station, TX 77843-3368
USA
e-mail: swamin@math.tamu.edu
FAX: (979) 845-6028

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Abstract

This is a compilation of the discussions among the participants of the AIM workshop on Day One. This discussion was moderated by David Farmer.

The discussion began with a solicitation for interesting open questions, with the partial intent of focussing the upcoming workshop days. Here are the questions that were posed, along with their authors.

1. Algorithmic aspects: Maurice Rojas *Discuss the algorithmic complexity aspects and applications of algorithmic complexity to the problem of finding zeros of random polynomials.* ⋄

2. Permanental polynomials: Yan Fyodorov *Interested in studying about permanental polynomials and permanents of characterisitic polynomials and their correlation functions. This has applicatoins to matching and counting. This also finds applications in graph theory , graph homomorphisms. Invariance under the symmetric group underlies these issues. There are no regular tool for dealing with permanents as both eigen values and eigen vectors are involved. Rojas comments that hyperbolic polynomials are used to approximate permanents.* ⋄

3. A-discriminants: Maurice Rojas *One should consider applications of A-discriminants one of whose special cases is the hyperdeterminant.* ⋄

4. Repulsion properties of zero sets: Yuval Peres *Understand the repulsion properties of zero sets of random polynomials. What is the general set up the study such questions. Determinantal processes have repulsion. If the discs are nearby nearly all analytic functions have repulsion. If discs are far apart only in special cases we see this. Shiffman adds that higher dimensional analogs should be studied. In fact with 3 polynomials attraction is noticed. Yuval comments that one should also consider repulsionof complex zeros of real polynomials. Ofer observes that there seems to be a larger gap near the real line. Rojas observes that this might be related to the Voorheve Wilder theorem.* ⋄

5. Concenteration methods Ofer Zeitouni *Concenteration methods are useful in studying random matrices. In the case of random polynomials difficulty arises as certain functions are not Lipschitz. Zelditch and Ofer mention that the major change in the case of random polynomials is that Lipschitz continuity is lost. Sodin questions about the setup for studying these questions. Are*

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we talking about real or complex zeros? These issues are related to hole probability. One has to fix the variables and let the degree vary. Ofer remarks that in random polynomials the closest zero to the unit circle goes to zero as the degree tends to infinity. Estimate how it approaches zero $\diamond$.

6. Guassian Free Fields: Amir Dembo *One has to study Gaussian Free fields and connnections to level sets of random polynomials. Oded makes remarks on lattices, graphs and Dirichlet inner products. One studies graphs that have certain singled out vertices called the boundary. One needs to study the space all functions zero over the boundary and real valued in the interior, equipped with the Dirichlet inner product.* $\diamond$

7. Extremal properties: Maurice Rojas and Michael Douglas *Extremal random functions of many variables should be studies, along with their gradient flows and basins of attraction. In particular small gradient flows for long time crops up in the study of inflationary models. Peres - Virag processes and random polynomial coefficients with brownian motion should also be studied.* $\diamond$

8. Random Beltrami fields: Richard Kenyon *Random Beltrami fields are also known as Injective analytic functions. One needs to look at nonconformal mappings, ϵ-small discs and elliptic function fields. There were comments from Balint and Sodin.* $\diamond$

9. Random Riemann surfaces: Michael Douglas *The moduli space of riemann surfaces has 3g-3 complex dimensions. This moduli space is equipped with natural petersson metric. This defines a probability measure. Does for large genus this probability measure concenterate? Consider hyperelliptic riemannian surfaces. Consult the work of Douglas, Witen and Aspinwall.* $\diamond$

10. Derivaties of random polynomials: Yuval Peres *Relationship between zeros of random polynomials and its derivatives needs to be studied.* $\diamond$

The discussion then concluded with the following question being posed to the audience: "Which topics would you like to see lectures on?" The answers, along with the people who suggested the topic, are listed below.

Lecture Topic A: Colomb gas models: Scott Sheffield *How are Colomb gas models and statistical physics models related to determinantal processes and random eigen vlaues. In Colomb gas change of measure arises naturally. Zelditch remarks that one needs to understand Gaussian measure in terms of zeros. Analogues of eigenvalues of random matrix face difficulty because of the rate functional and complex interactions.* $\diamond$

Lecture Topic B: Gaussian Free fields $\diamond$

Lecture Topic C: Motivations from physics: Zelditch *Motivations from physics such as from string theory and inflationary cosmological models should be discussed.* $\diamond$

Lecture Topic D: Random matrices as related to random polynomials: Manjunath Krishnapur and Balint Virag. $\diamond$