Analytical results

Group no. 2

A. Di Crescenzo, L. Gordillo, J. Mancilla
S. Marella, H. Riecke, L. Ward

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Stochastic dynamics of small networks of neurons
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A system of 3 differential equations

- For $t \geq 0$, we consider the system of differential equations

$$\frac{d}{dt}X(t) = -\frac{Z(t)}{\tau_r} - u\,X(t)$$
$$\frac{d}{dt}Y(t) = -\frac{Y(t)}{\tau_p} + u\,X(t)$$
$$\frac{d}{dt}Z(t) = \frac{Y(t)}{\tau_p} - \frac{Z(t)}{\tau_r}$$

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- Since $X(t) + Y(t) + Z(t) = 1$ we can reduce the system.

A system of 2 differential equations

- For $t \geq 0$, the new system is

$$\begin{pmatrix} \frac{d}{dt}X(t) \\ \frac{d}{dt}Y(t) \end{pmatrix} = \begin{pmatrix} \frac{1}{\tau_r} - u & \frac{1}{\tau_r} \\ u & -\frac{1}{\tau_p} \end{pmatrix} \begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} + \begin{pmatrix} -\frac{1}{\tau_r} \\ 0 \end{pmatrix}$$

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$$\begin{pmatrix} X(0) \\ Y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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- In our case

$$A = \begin{pmatrix} \frac{1}{\tau_r} - u & \frac{1}{\tau_r} \\ u & -\frac{1}{\tau_p} \end{pmatrix} \qquad B = \begin{pmatrix} -\frac{1}{\tau_r} \\ 0 \end{pmatrix}$$

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  - $\tau_p = 5$
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  - $\tau_p = 5$
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- Hence,

$$A = \begin{pmatrix} -0.19 & 0.01 \\ 0.20 & -0.20 \end{pmatrix}$$

- and then

$$e^{At} = \begin{pmatrix} \frac{4}{9}e^{-6t/25} + \frac{5}{9}e^{-3t/20} & -\frac{4}{9}e^{-6t/25} + \frac{4}{9}e^{-3t/20} \\ -\frac{5}{9}e^{-6t/25} + \frac{5}{9}e^{-3t/20} & \frac{5}{9}e^{-6t/25} + \frac{4}{9}e^{-3t/20} \end{pmatrix}$$

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$$X(t) = \frac{4}{9}e^{-6t/25} + \frac{5}{9}e^{-3t/20} + \frac{1}{54}\left(-3 + e^{-6t/25} + 2e^{-3t/20}\right)$$

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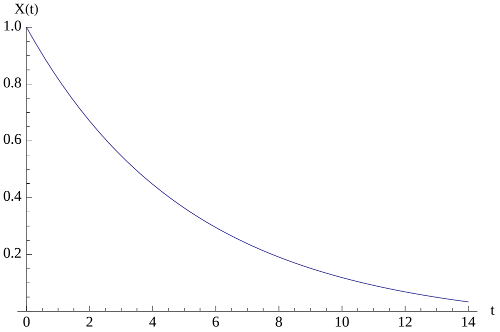
$$Y(t) = -\frac{5}{9}e^{-6t/25} + \frac{5}{9}e^{-3t/20} + \frac{1}{216}\left(-3 - 5e^{-6t/25} + 8e^{-3t/20}\right)$$

Figures

- Plot of $X(t)$

Figures

- Plot of $X(t)$



Figures

- Plot of $Y(t)$

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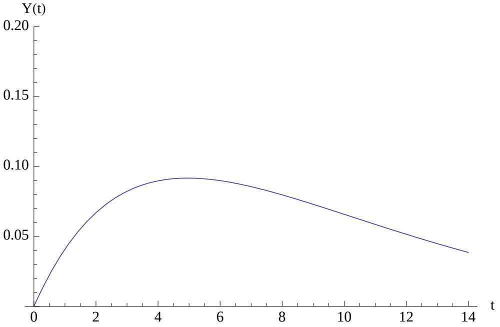

The end

Thank you for your attention