k-Schur functions

L Lapointe, University of Talca

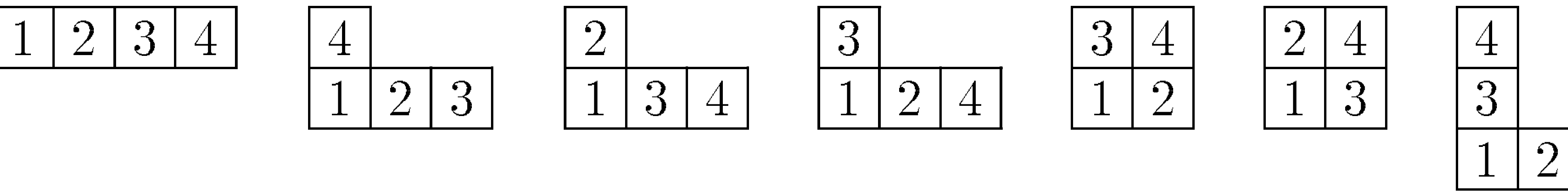
J Morse, University of Miami

q,t-Kostka polynomials: $K_{\mu\lambda}(q,t)$

$$J_\lambda[X;q,t] \;=\; \sum_\mu K_{\mu\lambda}(q,t)\, S_\mu$$

$$J_{2,2} \;=\; t^2 S_{(4)} + qt^2 S_{(3,1)} + qtS_{(3,1)} + tS_{(3,1)} + q^2t^2 S_{(2,2)} + S_{(2,2)} + q^2tS_{(2,1,1)} + qtS$$

$K_{\mu\lambda}(1,1) =$ Number of standard tableaux of shape μ



$$Map: \quad T \rightarrow q^{??}\, t^{??}\, S_{\text{shape}(T)}$$

An Observation

$$H_{1^4} = t^4\left(S_{22} + tS_{31} + t^2 S_{4}\right) + (t^2 + t^3)\left(S_{211} + tS_{31}\right) + \left(S_{1111} + tS_{211} + t^2 S_{22}\right)$$

$$H_{2,1,1} = t\left(S_{22} + tS_{31} + t^2 S_{4}\right) + (1 + qt^2)\left(S_{211} + tS_{31}\right) + q\left(S_{1111} + tS_{211} + t^2 S_{22}\right)$$

$$H_{2,2} = \underbrace{\left(S_{22} + tS_{31} + t^2 S_{4}\right)}_{S^{(2)}_{22}} + (tq + q)\underbrace{\left(S_{211} + tS_{31}\right)}_{S^{(2)}_{211}} + q^2\underbrace{\left(S_{1111} + tS_{211} + t^2 S_{22}\right)}_{S^{(2)}_{1111}}$$

Bigger positive basis for k-bounded Macdonalds

Coefficients remain positive sums of monomials

An Observation

$$H_{1^4} = t^4\left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + \left(t^2 + t^3\right)\left(S_{(2,1,1)} + tS_{(3,1)}\right) + \left(S_{(1,1,1,1)} + tS_{(2,1,1)} + t^2 S_{(2,2)}\right)$$

$$H_{2,1,1} = t\left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + \left(1 + qt^2\right)\left(S_{(2,1,1)} + tS_{(3,1)}\right) + q\left(S_{(1,1,1,1)} + tS_{(2,1,1)} + t^2 S_{(2,2)}\right)$$

$$H_{2,2} = \underbrace{\left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right)}_{S^{(2)}_{(2,2)}} + \left(tq + q\right)\underbrace{\left(S_{(2,1,1)} + tS_{(3,1)}\right)}_{S^{(2)}_{(2,1,1)}} + q^2\underbrace{\left(S_{(1,1,1,1)} + tS_{(2,1,1)} + t^2 S_{(2,2)}\right)}_{S^{(2)}_{(1,1,1,1)}}$$

Bigger positive basis for k-bounded Macdonalds

Coefficients remain positive sums of monomials

An Observation

$$H_{1^4} = t^4\left(S_{2,2} + tS_{3,1} + t^2 S_{4}\right) + \left(t^2+t^3\right)\left(S_{2,1,1} + tS_{3,1}\right) + \color{red}{\left(S_{1,1,1,1} + tS_{2,1,1} + t^2 S_{2,2}\right)}$$

$$H_{2,1,1} = t\left(S_{2,2} + tS_{3,1} + t^2 S_{4}\right) + \left(1+qt^2\right)\left(S_{2,1,1} + tS_{3,1}\right) + \color{blue}{q}\color{red}{\left(S_{1,1,1,1} + tS_{2,1,1} + t^2 S_{2,2}\right)}$$

$$H_{2,2} = \underbrace{\left(S_{2,2} + tS_{3,1} + t^2 S_{4}\right)}_{\color{red}{S^{(2)}_{2,2}}} + \left(tq+q\right)\underbrace{\left(S_{2,1,1} + tS_{3,1}\right)}_{\color{red}{S^{(2)}_{2,1,1}}} + \color{blue}{q^2}\underbrace{\color{red}{\left(S_{1,1,1,1} + tS_{2,1,1} + t^2 S_{2,2}\right)}}_{\color{red}{S^{(2)}_{1,1,1,1}}}$$

Bigger positive basis for k-bounded Macdonalds

Coefficients remain positive sums of monomials

An Observation

$$H_{1^4} = t^4\left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + (t^2+t^3)\left(S_{(2,1,1)} + tS_{(3,1)}\right) + \left(S_{(1,1,1,1)} + tS_{(2,1,1)} + t^2 S_{(2,2)}\right)$$

$$H_{2,1,1} = t\left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + (1+qt^2)\left(S_{(2,1,1)} + tS_{(3,1)}\right) + q\left(S_{(1,1,1,1)} + tS_{(2,1,1)} + t^2 S_{(2,2)}\right)$$

$$H_{2,2} = \left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + (tq+q)\left(S_{(2,1,1)} + tS_{(3,1)}\right) + q^2\left(S_{(1,1,1,1)} + tS_{(2,1,1)} + t^2 S_{(2,2)}\right)$$

$$\underbrace{\qquad\qquad}_{S^{(2)}_{(2,2)}} \qquad \underbrace{\qquad\qquad}_{S^{(2)}_{(2,1,1)}} \qquad \underbrace{\qquad\qquad}_{S^{(2)}_{(1,1,1,1)}}$$

Bigger positive basis for k-bounded Macdonalds

Coefficients remain positive sums of monomials

New basis? $S_{\lambda}{}^{(k)}$

Indexed by partitions with no part larger than k

The atoms:

$$\left(\begin{array}{|c|c|}\hline 3&4\\\hline 1&2\\\hline\end{array}\;\begin{array}{|c|cc}\hline 4&\multicolumn{2}{c}{}\\\hline 1&\multicolumn{1}{|c|}{2}&\multicolumn{1}{c|}{3}\\\hline\end{array}\;\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\\hline\end{array}\right)\left(\begin{array}{|c|c}\hline 4&\\\cline{1-1} 3&\\\hline 1&\multicolumn{1}{c|}{2}\\\hline\end{array}\;\begin{array}{|c|cc}\hline 3&\multicolumn{2}{c}{}\\\hline 1&\multicolumn{1}{|c|}{2}&\multicolumn{1}{c|}{4}\\\hline\end{array}\right)\left(\begin{array}{|c|c}\hline 4&\\\cline{1-1} 2&\\\hline 1&\multicolumn{1}{c|}{3}\\\hline\end{array}\;\begin{array}{|c|cc}\hline 2&\multicolumn{2}{c}{}\\\hline 1&\multicolumn{1}{|c|}{3}&\multicolumn{1}{c|}{4}\\\hline\end{array}\right)\Bigg(\cdots$$

<u>Map:</u> $T \longrightarrow t^{\mathrm{charge}(T)} S_{\mathrm{shape}(T)}$

$$H_{1^4} = t^4\left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + t^2\left(S_{(2,1,1)} + tS_{(3,1)}\right) + t^3\left(S_{(2,1,1)} + tS_{(3,1)}\right)\cdots$$

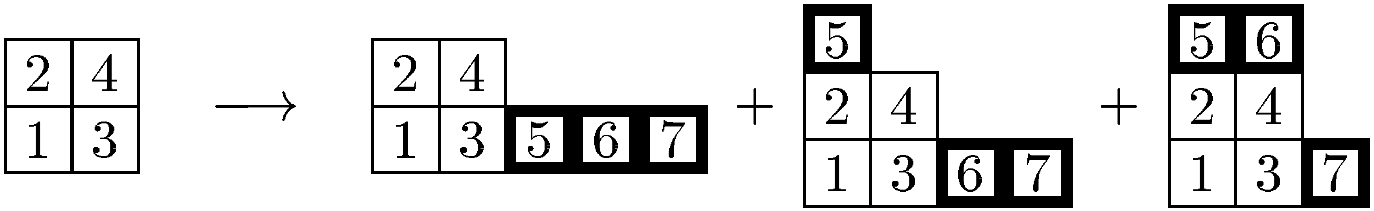
$$H_{2,1,1} = t\left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + \left(S_{(2,1,1)} + tS_{(3,1)}\right) + qt^2\left(S_{(2,1,1)} + tS_{(3,1)}\right)\cdots$$

$$H_{2,2} = \left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + tq\left(S_{(2,1,1)} + tS_{(3,1)}\right) + q\left(S_{(2,1,1)} + tS_{(3,1)}\right) + \cdots$$

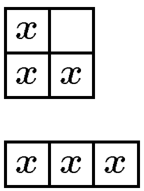
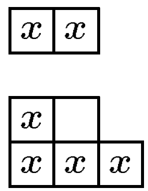
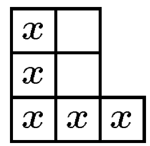
ATOM: $\mathbb{A}_\lambda^{(k)} = \mathbb{K}_{\lambda\to k}\,\mathbb{S}_{\lambda_1}\,\mathbb{A}_{\lambda_2,\dots,\lambda_n}^{(k)} \qquad \lambda_1 \le k$

1. Promotion operators: $\mathbb{S}_{\lambda_1}$

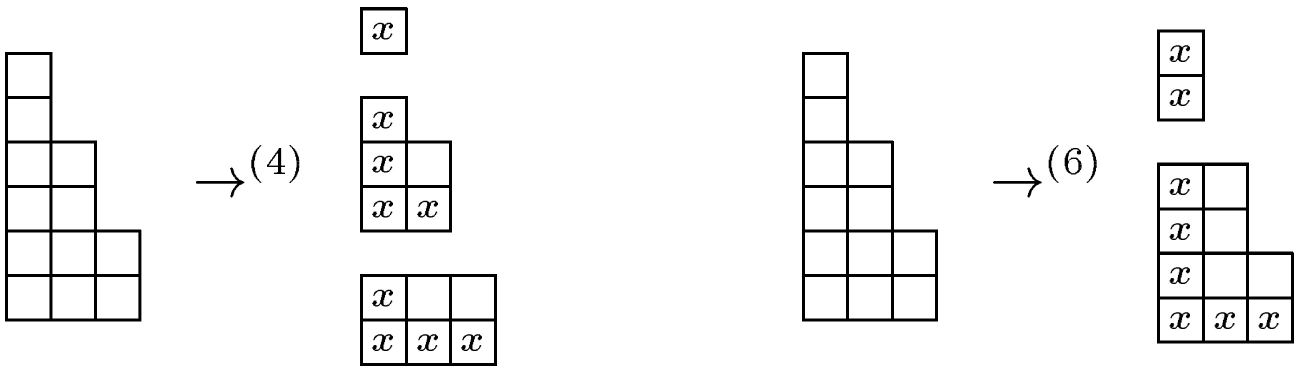
Add horizontal λ_1-strip



2. Filtering operator: $\mathbb{K}_{\lambda\to k}$

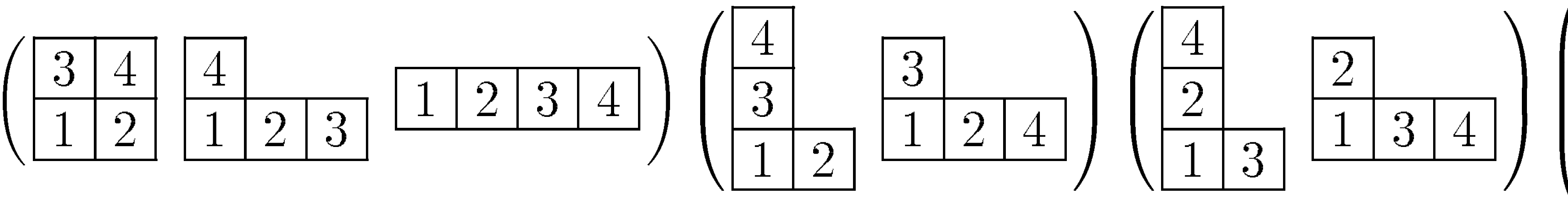
Kill T unless  can be extracted from T (or  or )

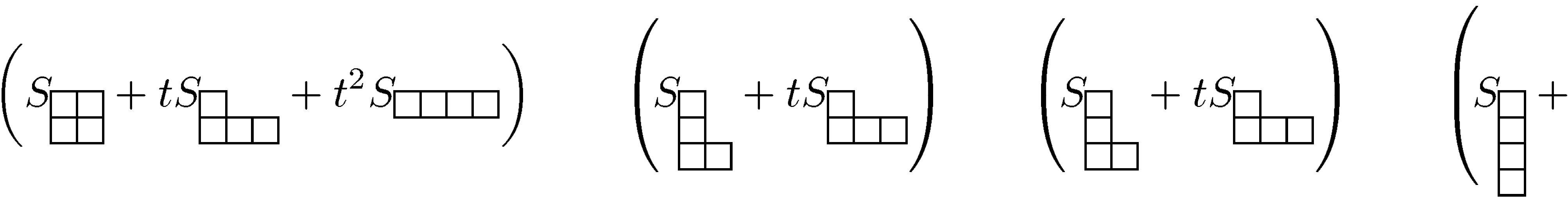
k-split: $\lambda^{\to(k)}$



Atoms

- Partition the set of tableaux (depending on λ and k)





- Conjecture: form a basis for $\mathcal{L}\{J_\lambda\}_{\lambda_1 \le k}$

- Conjecture: expansion coefficients refine q, t-Kostkas

$$J_\lambda = \sum_\mu K_{\mu\lambda}^{(k)}(q,t)\, A_\mu^{(k)} \quad \text{where} \quad K_{\mu\lambda}^{(k)}(q,t) \in \mathbb{N}[q,t]$$

A refinement for Schur functions: $t = 1$

- Involution: $\omega S_\lambda^{(k)} = S_{\lambda^{\omega_k}}^{(k)}$

- k-Pieri rule:

$$h_\ell \, S_\lambda^{(k)} = \sum_{\mu \in \mathcal{C}_{\lambda,\ell}^{(k)}} S_\mu^{(k)} \quad \text{where} \quad \mathcal{C}_{\lambda,\ell}^{(k)} \in \mathbb{N}$$

- k-Kostka Numbers:

$$h_\lambda = \sum_\mu K_{\mu\lambda}^{(k)} \, S_\mu^{(k)} \quad \text{where } K_{\mu\lambda}^{(k)} \in \mathbb{N}$$

- k-Littlewood Richarsdon rule

$$S_\lambda^{(k)} S_\mu^{(k)} = \sum_\nu c_{\lambda\mu}^{\nu,k} \, S_\nu^{(k)} \quad \text{where } c_{\lambda\mu}^{\nu,k} \in \mathbb{N}$$

k-***Schurs:*** $S^{(k)}_{\lambda} = K^{(k)}_{\lambda_1} \, S_{\lambda_1} \, S^{(k)}_{\lambda_2,\ldots,\lambda_\ell}$ ***where*** $\lambda_1 \leq k$

1. Multiply by S_{λ_1}: $$S_3 \, S^{(5)}_{2,2} = S_3 \, S_{2,2} = S_{3,2,2} + S_{4,2,1} + S_{5,2}$$

2. Apply projection operator: $K^{(k)}_{\lambda_1}$

- Expand in terms of k-split basis: P^k_μ $$S_3 \, S^{(5)}_{2,2} = P^5_{3,2,2} + P^5_{4,2,1} - P$$

$$P^4_{(3,3,2,2,1,1)} = S_{(3,3)} \, S_{(2,2,1)} \, S_{(1)} \qquad P^5_{(4,2,1)} = S_{(4,2)} \, S_{(1)}$$

- Kill P^k_μ unless $\mu_1 = \lambda_1$: $$S_3 \, S^{(5)}_{2,2} = P^5_{3,2,2} = S_3$$

k-Schur functions

$$\mathcal{L}\left\{J_\lambda\right\}_{\lambda_1\le k} = \mathcal{L}\left\{H_\lambda\right\}_{\lambda_1\le k} = \mathcal{L}\left\{S_\lambda^{(k)}\right\}_{\lambda_1\le k}$$

$$J_\lambda = \sum_\mu K_{\mu\lambda}^{(k)}(q,t) S_\mu^{(k)} \quad \text{where} \quad K_{\mu\lambda}^{(k)}(q,t) \in \mathbb{Z}[q,t]$$

$$S_\lambda^{(k)} = S_\lambda \quad \text{for large } k$$

$$S_\lambda^{(k)} = S_\lambda + \text{larger terms}$$

$$S_R\, S_\lambda^{(k)} = S_{R,\lambda}^{(k)}$$

Positivity, involution, etc. for $k = 2$

HMMMMMM

$$H_{1^4} = t^4\left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + \left(t^2 + t^3\right)\left(S_{(2,1,1)} + tS_{(3,1)}\right) + \left(S_{(1,1,1,1)} + tS_{(2,1,1)} + t^2 S_{(2,2)}\right)$$

$$H_{2,1,1} = t\left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + \left(1 + qt^2\right)\left(S_{(2,1,1)} + tS_{(3,1)}\right) + q\left(S_{(1,1,1,1)} + tS_{(2,1,1)} + t^2 S_{(2,2)}\right)$$

$$H_{2,2} = \left(S_{(2,2)} + tS_{(3,1)} + t^2 S_{(4)}\right) + \left(tq + q\right)\left(S_{(2,1,1)} + tS_{(3,1)}\right) + q^2\left(S_{(1,1,1,1)} + tS_{(2,1,1)} + t^2 S_{(2,2)}\right)$$

Study $q = t = 1$:

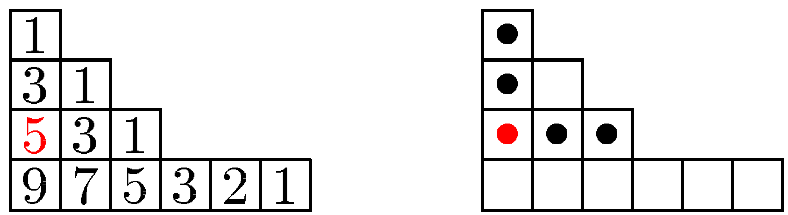
$$h_{1,1,1,1} = 1\left(S_{(2,2)} + S_{(3,1)} + S_{(4)}\right) + 2\left(S_{(2,1,1)} + S_{(3,1)}\right) + 1\left(S_{(1,1,1,1)} + S_{(2,1,1)} + S_{(2,2)}\right)$$

Pieri Rule $\Longrightarrow$ Kostka Numbers $\Longrightarrow$ q, t-Coefficients

k-tableaux

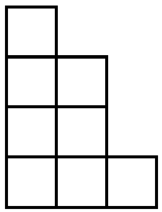
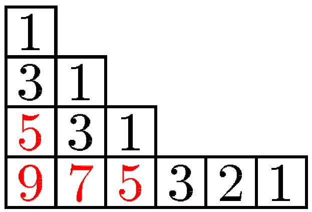
Bijection: k-bounded partitions $\leftrightarrow$ $k+1$-core diagrams

$k = 3$:



k-tableaux

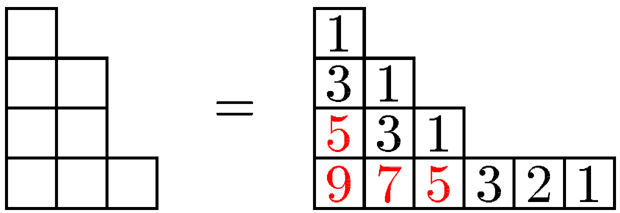
Bijection: k-bounded partitions $\leftrightarrow$ $k+1$-core diagrams

$k = 3 :$  $=$ 

k-tableaux

Bijection: k-bounded partitions $\leftrightarrow$ $k+1$-core diagrams

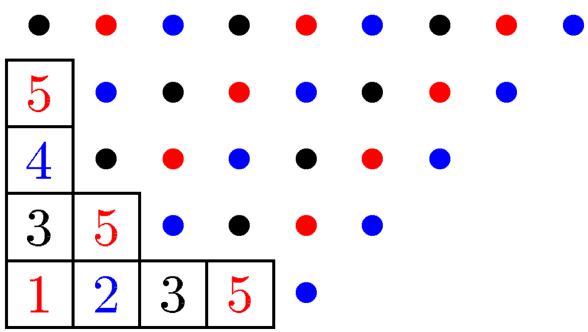
$k = 3:$

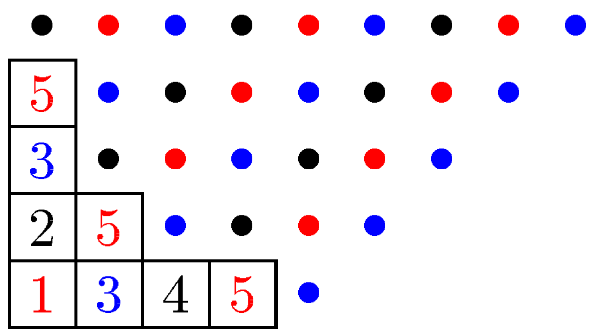


Standard k-Tableaux:

$k+1$-core filled strictly increasing in rows and columns

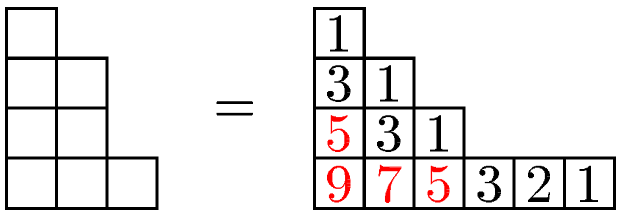
Multiplicities have same color ($k+1$-residue)





k-tableaux

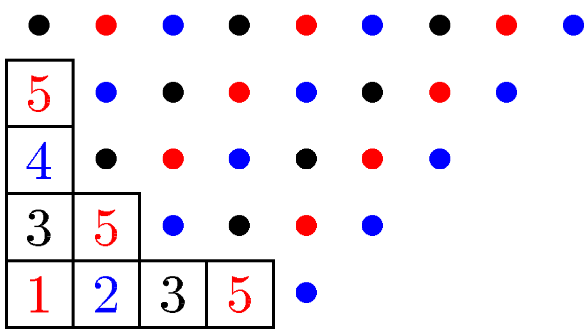
Bijection: k-bounded partitions $\leftrightarrow$ $k+1$-core diagrams

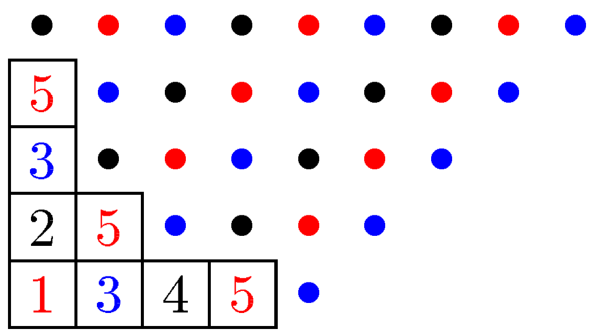
$k = 3:$ 

Standard k-Tableaux:

$k+1$-core filled strictly increasing in rows and columns

Multiplicities have same color ($k+1$-residue)





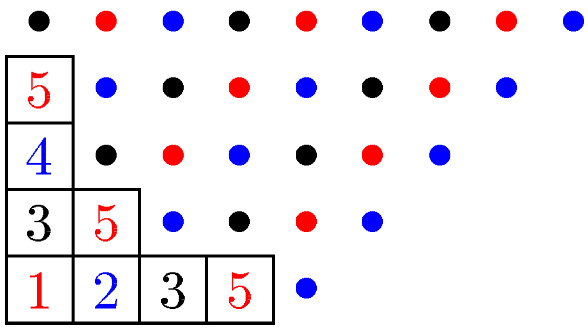
reduced words for $\tilde{S}_{k+1}/S_{k+1}$: 5 4 3 2 1 = $s_0\, s_2\, s_1\, s_2\, s_0$

Characterize k-Schur functions

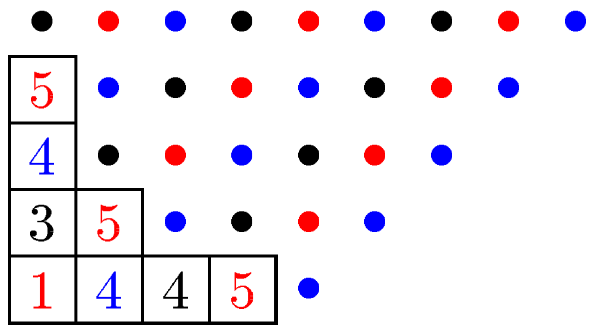
Semi-Standard k-Tableaux:

Column strict filling of $k+1$-core

Exactly α_i colors for letter i



weight: (1,1,1,1,1)



(1,0,1,2,1)

Property:

$$K^{(k)}_{\mu\lambda} = \#k\text{-tableaux of shape } \mu \text{ and weight } \lambda = \begin{cases} 0 & if \quad \mu \lhd \lambda \\ 1 & if \quad \mu = \lambda \end{cases}$$

Characterize k-Schurs by inverting expression

$$h_\lambda = \sum_{\mu \trianglerighteq \lambda} K^{(k)}_{\mu\lambda} \, S^{(k)}_\mu$$

k-Schur properties

- Basis for $\Lambda^{(k)} = \mathcal{L}\{h_\lambda\}_{\lambda_1 \le k}$
- Involution: $\omega S^{(k)}_\lambda = S^{(k)}_{\lambda^{\omega_k}}$ where $\lambda^{\omega_k} = core(\lambda)'$.
- k-Pieri rule:

$$h_\ell \, S^{(k)}_\lambda = \sum_{\mu \in \mathcal{C}^{(k)}_{\lambda,\ell}} S^{(k)}_\mu \quad \text{where} \quad \mathcal{C}^{(k)}_{\lambda,\ell} = \{\mu \mid \mu = \lambda \; + \; colored\ corners\ \}$$

- k-Kostka numbers

$$h_\lambda = \sum_{\mu \ge \lambda} K^{(k)}_{\mu\lambda} \, S^{(k)}_\mu \quad \text{where} \quad K^{(k)}_{\mu\lambda} = \#\, k\text{-tableaux shape } \mu$$

- Dual basis (for $\Lambda/\mathcal{J}$) given by

$$\mathfrak{S}_\lambda = \sum_{T = ktab\ shape\ core(\lambda)} x^T$$

- Cauchy Kernel

$$\sum_{\lambda_1 \le k} S^{(k)}_\lambda[X] \mathfrak{S}^{(k)}_\lambda[Y] = \prod_{i,j} \frac{1}{(1 - x_i y_j)} \mod \mathcal{J}$$

q, t-Kostka polynomials

k-Kostka numbers:

$$h_\lambda = \sum_{\mu \geq \lambda} K_{\mu\lambda}^{(k)} \, S_\mu^{(k)} \quad \text{where } K_{\mu\lambda}^{(k)} = \#k\text{–tableaux}$$

q, t-generalize

$$H_\lambda[X; q, t] = \sum_{\mu \geq \lambda} K_{\mu 1^n}^{(k)}(q, t) \, S_\mu^{(k)}$$

where $K_{\mu\lambda}^{(k)}(1,1) = \#$ standard k–tableaux = # reduced words

$$H_{2,2}[X; q, t] = S_{(2,2)}^{(2)} + (tq + q) \, S_{(2,1,1)}^{(2)} + q^2 S_{(1,1,1,1)}^{(2)}$$

$$\begin{array}{|c|}\hline 4 \\ \hline 3 \\ \hline \end{array} \atop \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline \end{array} \qquad \begin{array}{|c|}\hline 4 \\ \hline 2 \\ \hline \end{array} \atop \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline \end{array} \qquad \leftrightarrow \qquad s_0 \, s_3 \, s_1 \, s_2 = s_0 \, s_1 \, s_3 \, s_2$$

Quantum cohomology: $QH^*(Gr_{a,n})$

Enumeration of "Gromov-Witten invariants"

- Additive structure: $QH^*(Gr_{a,n}) = H^*(Gr_{a,n}) \otimes \mathbb{Z}[q]$

  Schubert classes σ_λ for $\lambda \subseteq a \times (n-a)$ are a basis

- Product:

$$\sigma_\lambda * \sigma_\mu = \sum_{\substack{\nu \subseteq a \times (n-a) \\ |\nu| = |\lambda| + |\mu| - dn}} q^d \, C_{\lambda\mu}^{\nu,d} \, \sigma_\nu$$

$C_{\lambda\mu}^{\nu,d} = \#$ rational curves of degree d in $Gr_{a,n}$ that meet fixed generic translates of the Schubert varieties X_λ, X_μ and $X_{\hat{\nu}}$

Symmetric Function Connection:

$QH^*(Gr_{a,n}) \cong \Lambda \otimes \mathbb{Z}[q]/\mathcal{I}_q$ where $\mathcal{I}_q = \langle e_{n-a+1}, \ldots, e_{n-1}, e_n + (-1)^a q \rangle \cup \{h_i : i >$

$$\sigma_\lambda \leftrightarrow S_\lambda \quad \text{for} \quad \lambda \subset a \times (n-a)\,.$$

Structure constants

Cohomology structure: $\sigma_\lambda * \sigma_\mu \quad \leftrightarrow \quad S_\lambda\, S_\mu \mod \mathcal{I}_q$

$$S_\lambda S_\mu = \sum_\nu c^\nu_{\lambda\mu} S_\nu$$

$$\downarrow$$

$$S_\lambda S_\mu \mod \mathcal{I}_q = \sum_{\nu\subset a\times(n-a)} ???\, S_\nu$$

$$\updownarrow$$

$$\sigma_\lambda * \sigma_\mu = \sum_{\nu\subset a\times(n-a)} ???\, \sigma_\nu$$

Structure constants

Cohomology structure: $\sigma_\lambda * \sigma_\mu \quad \leftrightarrow \quad S_\lambda\, S_\mu \mod \mathcal{I}_q$

$$S_\lambda S_\mu = \sum_{\nu \subset a\times(n-a)} c^\nu_{\lambda\mu} S_\nu + \sum_{\nu \not\subset a\times(n-a)} c^\nu_{\lambda\mu} S_\nu$$

$\downarrow$

Problem: $S_\lambda \not\equiv 0$ for $\lambda \not\subset a \times (n-a)$

$$S_\lambda S_\mu \mod \mathcal{I}_q = \sum_{\nu \subset a\times(n-a)} c^\nu_{\lambda\mu}\, S_\nu + \sum_{\hat\nu \subset a\times(n-a)} c^{\hat\nu}_{\lambda\mu}\,(\pm q^*)\, S_{\hat\nu}$$

$\updownarrow$

$$\sigma_\lambda * \sigma_\mu = \sum_{\nu \subset a\times(n-a)} ???\ \sigma_\nu$$

Structure constants

Cohomology structure: $\sigma_\lambda * \sigma_\mu \quad \leftrightarrow \quad S_\lambda\, S_\mu \mod \mathcal{I}_q$

$$S^{(k)}_\lambda\, S^{(k)}_\mu = \sum_{\nu=(k+1,a)-diagram} c^{\nu,k}_{\lambda\mu}\, S^{(k)}_\nu + \sum_{\nu\neq(k+1,a)-diagram} c^{\nu,k}_{\lambda\mu}\, S^{(k)}_\nu$$

$$\downarrow$$

$$S_\lambda\, S_\mu \mod I_q = \sum_{\nu=(k+1,a)-diagram} c^{\nu,k}_{\lambda\mu}\, S_\nu$$

$$\downarrow$$

$$S_{\hat\lambda}\, S_{\hat\mu} \mod \mathcal{I}_q = \sum_{\hat\nu\subset a\times(k+1-a)} q^d\, c^{\nu,k}_{\lambda\mu}\, S_{\hat\nu}$$

$$\downarrow$$

$$\sigma_{\hat\lambda} * \sigma_{\hat\mu} = \sum_{\hat\nu\subset a\times(k+1-a)} q^d\, c^{\nu,k}_{\lambda\mu}\, \sigma_{\hat\nu}$$

Gromov-Witten invariants = k-Littlewood Richardson coefficients

Alternate k-Schurs

- Schur functions in symmetric function theory of $\Lambda^{(k)}$

  Conjecture: k-q,t-Kostka polynomials lie in $\mathbb{N}[q,t]$.

  Conjecture: (dual) k-Schurs are symmetric component of affine Schubert polynomials

- Schur functions in theory of quantum cohomology

  Conjecture: k-Schur functions are Schubert basis of the homology of the affine Grassmannian

  Conjecture: dual k-Schurs are Schubert basis of the cohomology of the affine Grassmannian

Another characterization

Want definition for k-Schurs:

$$S_\lambda^{(k)} = \sum_{some\ T} t^{stat(T)} x^T$$

Want:

- Affine Pieri rule for dual k-Schurs

$$\sum_{T=ktab} x^T$$

- Affine Schensted
- Affine Plactic Monoid
- Tie together various characterizations for k-Schurs

Strong order k-tableaux

Chain in Bruhat (strong) order:

Sequence of $k+1$-cores:

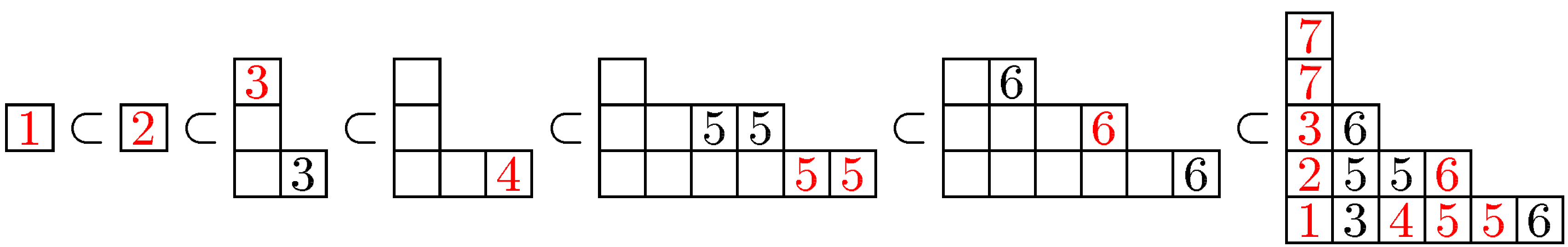
$$\emptyset \subset \gamma^{(1)} \subset \gamma^{(2)} \subset \ldots \subset \gamma^{(\ell)} = \gamma$$

k-bounded hooks in $\gamma^{(i)}$ = # k-bounded hooks in $\gamma^{(i+1)} - 1$

Dual Standard Case:

Filling of a $k+1$-core γ with letter i in cells of $\gamma^{(i+1)}/\gamma^{(i)}$

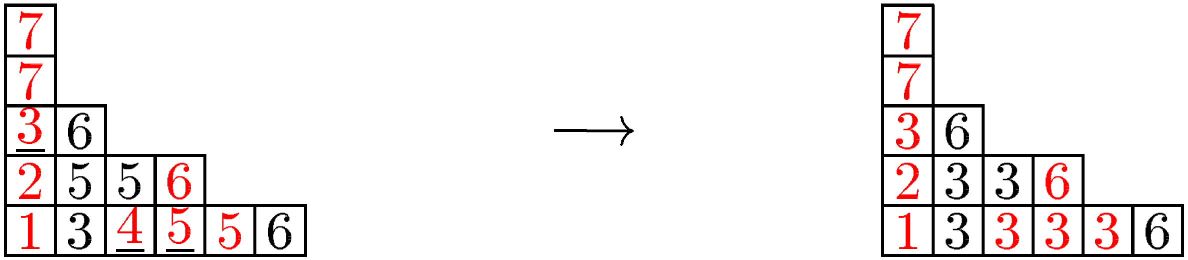
One ribbon containing letter i is marked



Semi-Standard Dual

Put letters $i(i+1)\cdots(i+\ell) \longrightarrow a$

IF NW cell of the marked ribbons containing letters $i(i+1)\cdots(i+\ell)$ read increasing



Weight: α_i = number of marked ribbons containing i

Definition

$$S_\lambda^{(k)}[X;t] = \sum_{\substack{T=dual\ k\ tab \\ of\ shape\ core(\lambda)}} t^{stat(T)} x^T$$

where

$$stat(T) = \sum_{i=1}^{n} t^{a(spin(i))+b}$$

$a = \#$ of ribbons containing letter i

$b = \#$ of ribbons containing letter i above marked one

Future work

- Combinatorial interpretation for k-Littlewood Richardson coefficients and Macdonald coefficients
- affine RSK insertion algorithm and plactic monoid
- (dual) k-Schur functions = Schubert classes in (co)homology of the loop Grassmannian
- Affine Schubert polynomials that reduce to k-Schur functions in the grassmannian
- Unimodality, sieved sum identities
- Representation theoretic interpretation for k-Schur functions