ARTICLES ON GENERALIZED KOSTKA POLYNOMIALS

The following references discuss various aspects of generalized Kostka polynomials.

(1) Hatayama, Goro; Kuniba, Atsuo; Okado, Masato; Takagi, Taichiro; Tsuboi, Zengo. "Paths, crystals and fermionic formulae." *MathPhys odyssey*, 2001, 205–272, *Prog. Math. Phys.* **23**, Birkhäuser Boston, Boston, MA, 2002.

(2) Hatayama, G.; Kuniba, A.; Okado, M.; Takagi, T.; Yamada, Y. "Remarks on fermionic formula." *Recent developments in quantum affine algebras and related topics* (Raleigh, NC, 1998), 243–291, *Contemp. Math.* **248**, Amer. Math. Soc., Providence, RI, 1999.

The above two papers define generalized Kostka polynomials from the point of view of crystal theory. In this setting generalized Kostka polynomials and their unrestricted and level-restricted counterparts are one-dimensional configuration sums. The sums run over elements in the tensor products of Kirillov-Reshetikhin crystals and give combinatorial formulas for q-deformations of tensor product multiplicities with energy statistic. In these papers, the existence of Kirillov-Reshetikhin crystals is conjectured for general types and explicit fermionic formulas are conjectured.

Here is the description of the second paper from MathSciNet: This paper is an extension of [the first paper] in which fermionic formulas and crystal bases associated with all quantum affine algebras $U_q(X_N^{(r)})$ are conjectured; in the previous paper only the nontwisted case $r = 1$ was considered. This paper also contains a comprehensive bibliography and a very detailed introduction on the history of the subject.

Fermionic formulas are q-polynomials or infinite q-series that are sums of products of q-binomial coefficients. The specific formulas depend on the root structure of the algebra $X_N^{(r)}$. They originate in the Bethe Ansatz study of exactly solvable lattice models.

The authors conjecture the existence of a family of finite-dimensional affine crystal bases $B^{a,i}$ associated with the finite-dimensional irreducible $U_q'(\mathfrak{g})$ modules $W_i^{(a)}$. The $W_i^{(a)}$ are also called Kirillov-Reshetikhin modules. Let $\overline{\mathfrak{g}}$ be a classical subalgebra of $\mathfrak{g}$. Then as a $U_q(\overline{\mathfrak{g}})$-module, $W_i^{(a)}$ decomposes into the highest weight component $V(i\Lambda_a)$ plus terms of weight strictly less than $i\Lambda_a$. Here Λ_a is the fundamental weight of $\overline{\mathfrak{g}}$.

The corner-transfer-matrix method of statistical mechanics provides the definition of one-dimensional configuration sums $X(B, \lambda, q)$ depending on a tensor product of finite crystals $B = B^{a_1,i_1} \otimes \cdots \otimes B^{a_L,i_L}$, called a path, and a dominant weight λ. Due to the equivalence of the corner-transfer-matrix

method and the Bethe Ansatz, it is conjectured that $X(B, \lambda, q)$ is equal to the corresponding fermionic formula $M(B, \lambda, q)$.
In the limit of semi-infinite paths, the authors obtain spinon character formulas and Lepowsky-Primc type conjectural formulas for vacuum string functions. It is shown that at $q = 1$ the fermionic formulas satisfy certain recursion relations called a Q-system which can be solved in terms of characters of the various classical subalgebras.

(3) A. Schilling, S.O. Warnaar. "Inhomogeneous lattice paths, generalized Kostka polynomials and A_{n-1} supernomials." *Commun. Math. Phys.* **202** (1999), 359–401.
This paper defines generalized Kostka polynomials labelled by a partition and a sequence of rectangles. Interpretation of these polynomials in terms of crystal paths with energy statistic, Littlewood-Richardson tableaux and charge statistic, and fermionic formulas are given.

(4) A.N. Kirillov, A. Schilling, M. Shimozono, "A bijection between Littlewood-Richardson tableaux and rigged configurations." *Selecta Mathematica (N.S.)* **8** (2002), 67–135.
Here a proof of the fermionic formula for the generalized Kostka polynomials of type A is given. The proof uses a bijection between Littlewood-Richardson tableaux and rigged configurations that preserves the statistic.

(5) A. Schilling, M. Shimozono, "Fermionic formulas for level-restricted generalized Kostka polynomials and coset branching functions." *Commun. Math. Phys.* **220** (2001), 105–164.
This paper deals with fermionic formulas for level-restricted generalized Kostka polynomials of type A. It is shown that the effect of level-restriction on rigged configurations is a modification of the vacancy numbers.

(6) A. Schilling, "q-Supernomial coefficients: From riggings to ribbons," in: *MathPhys Odyssey* 2001, M. Kashiwara and T. Miwa (eds.), Birkhäuser Boston, Cambridge, MA, 2002, 437–454.
In this paper, fermionic formulas for unrestricted Kostka polynomials of type A by Hatayama et al. are related to ribbon tableaux generating functions weighted by cospin. This is done by interpreting the fermionic formulas in terms of "ribbon" rigged configurations and providing a bijection to ribbon tableaux.

(7) A. Schilling, "Crystal structure on rigged configurations," (to appear on math arXiv).
This paper deals with rigged configurations for unrestricted Kostka polynomials for type ADE and defines a crystal structure on this set. The rigged configurations in this paper are different from the "ribbon" rigged configurations; they are more closely linked to the definition of one-dimensional configuration sums in terms of crystal paths and energy statistic.