June 7, 2007

Open problem session

1. Tamas Hausel

$$\Omega_{g,k} = \sum_{\mu} \tilde{H}_\mu(\underline{x}_1) \cdots \tilde{H}_\mu(\underline{x}_k)\, H_\mu(z,w)$$

Q: i) Can one express the $\tilde{H}_\mu$ in terms of the $\Omega_{g,k}$?

ii) To what extent is $\Omega_{g,k}$ a TFT?
(in that case everything determined by )

 ← $k=3$, $g=0$

No? you can't distinguish $\tilde{H}_\mu$ from conjugate

$k=2$, $g=0$ Cauchy kernel, orthogonality of $\tilde{H}_\mu$

2. Nick Proudfoot

M = multiplicative quiver variety

Q: Is M homotopic to a projective variety in a natural way?

E.g.  1 $\quad M = \{ z, w : zw + 1 \neq 0 \}$

$\wr\wr$

<u>Idea</u>: M's are de Rham spaces

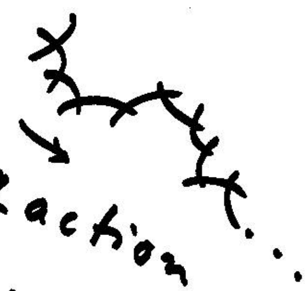
topologically $\longrightarrow$ $\wr\wr$

Dolbeault spaces

/ homotopic

Lagrangian core

<u>context</u>: $\mathbb{C}^\times$ actions on symplectic mflds & their Morse theory

universal cover $\tilde{M} \simeq$ 

$\mathbb{Z}$ action

<u>model</u>: Tate construction of elliptic curves.

<u>class of examples</u> $\quad Q$ with $\alpha = 1, \ldots, 1$

<u>Another context</u>: Springer theory $\rightsquigarrow$ Higgs bundles $= M_{Dol}$

$\{ n, \text{flag} \mid n \text{ preserves flag} \} = T^*G/B = \tilde{\mathcal{N}}$

$\downarrow$

$n \in \mathcal{N} \subset \mathfrak{g}$

$\swarrow$

flag $\in G/B$

For $G((t))$ & flags $\rightsquigarrow$ affine flags

Q: Hyperkähler structure on multiplicative quiver varieties (starshaped ok)

$\{1+zw \neq 0\}$ has $\omega = \dfrac{dz \wedge dw}{1+zw}$

hol symplectic ... is this hyperkähler?

Tian-Yau $\overset{?}{\Rightarrow}$ hyperkähler structure on $\{1+zw \neq 0\}$

3. Q: Find a good compactification of character varieties,

normal crossing
spread out-able
Poisson structure preserved?

& use it to calculate H^*

X symplectic $\Rightarrow$ $f, g \mapsto df, g \overset{\omega}{\longmapsto}$ vector field

[M symplectic since H^1(Riemann surface) has symplectic form]
M ← character variety

Another context: character varieties are cluster varieties (Fock-Goncharov)

Problem With at least one puncture variety is $\prod Gl_n$'s
← rich Poisson structure

unless we fix conjugacy classes

suggestion Compactify $\prod G/u$!

compatible with TFT

4. N. Katz

$$\prod_i [A_i, B_i] \prod_j E_j \cdot D_j \cdot E_j^{-1} = 1$$

$M_{\{D_j\}} = \{ A_i, B_i, E_i : \quad \}$

counting points for this variety easily relates to counting for $M_{\{C_j\}}$

Q: Count points on this space instead of on (tricky) quotients?

5. N. Katz

Q: Is there an algorithm to determine solvability of the multiplicative Deligne Simpson, write single irred solution, or parameterize all, explain them? Are solutions generated by "minimal" ones in comprehensible way?

Crawley-Boevey Indecomp. repn of quivers (non-Dynkin, or extended Dynkin) are wild problems (in some sense) $\in$ irred solns of additive Deligne Simpson

Kac theorem context: write down a single indecomposable for given imaginary root

N.B. real roots $\leftrightarrow$ rigid solns

Problem w/ minimality: may need to increase n in the process of reducing to minimal solution.

6. F.R. Villegas

$$\sum_{n \geq 0} \# \mathrm{Hom}(\pi_1(\Sigma_g), S_n) \frac{x^n}{n!}$$

$$= \prod_{n \geq 1} (1 - x^n)^{-v_n}$$

"$\mathbb{F}_1$ version" of character variety

$v_n \in \mathbb{Z}$ $(g > 0)$

what is v_n? (is it > 0, what does it count?). we know

$$\sum_{d | n} d v_d = \# \{ \Gamma < \pi_1(\Sigma_g) \}_{\text{index } n}$$

suggests "Frobenius action on $\{\Gamma < \pi_1(\Sigma_g)\}$"

$v_d = \#$ "closed points" (Frob orbits) what about?

$$\sum_{n\geq 0} \# \mathrm{Hom}(\pi_1(\Sigma_g), Gl_n(\mathbb{F}_q)) \frac{x^n}{n!}$$

$$= \prod (1-x^n)^{-\nu_n(q)}$$

Do we know what $\nu_n(q)$ is?

7. D. Ben-Zvi

Q If $G \neq Gl_n$ does $G^\vee$ appear?

Sl_n conjecture: E-polynomial of smooth $Sl_n\mathbb{C}$ character variety = stringy E-polynomial for PGl_n"

ok Slp? $\Rightarrow$ relations between char of Sl_n, PGl_n?

Remark (G. Lehrer)

Sl_n, $Gl_n / \mathbb{F}_q$

$\# Sl_n(\mathbb{F}_q)/Gl_n(\mathbb{F}_q) = \# PGl_n(\mathbb{F}_q)/Gl_n(\mathbb{F}_q)$

not that trivial, ad hoc proof

types don't match

Similar statements for other Langlands dual pairs.

A. Ram Lusztig classification

char of $G(\mathbb{F}_q) \overset{\cdots\cdots}{\underset{\text{rough}}{\longleftrightarrow}}$ conj classes in $G^\vee(\mathbb{F}_q)$

Interpret as Langlands duality?

8. <u>Oblomkov</u>

Action of mapping class group on H^*(char varieties)?

Hook polynomial $\dashrightarrow$ zeta fctn of curve

9. D. <u>Nadler</u> What is the relation with work of Ngo? (Counting points on $\mathcal{M}_{Higgs}$ solved fundam. lemma using Hitchin system)

10. <u>N. Katz</u> $X, Y \,/\, \mathbb{Z}[\frac{1}{N}]$ projective smooth suppose for all primes $p \nmid N$

$\# X(\mathbb{F}_p) = \# Y(\mathbb{F}_p) \Rightarrow$ same zeta fctn for each such p $\Rightarrow X, Y$ have same $h^{p,q}$!

<u>Q</u> How can we deduce $h^{p,q}$'s from these numbers?

(assume X, Y have geometrically connected fibers, to be safe)

$\# X(\mathbb{F}_p) \sim p^{\dim} + \cdots$

$\Rightarrow \dim =$ largest # s.t. $\frac{\# X(\mathbb{F}_p)}{p^{\dim}} \to 0$

e.g. suppose $d = 1$ $\# X(\mathbb{F}_q) = p + 1 - a_p$

$$\limsup \left|\frac{a_p}{\sqrt{p}}\right| \leq 2g$$

Q: Is this an equality ?

If ℓ-adic repn on H^1 is big (open in GSp_{2g}) & we believe a "Sato-Tate" distribution then Frob close to Id often and get equality.

True for $g=1$. But what if limsup $=2$ does this force $g=1$?