A SOFTWARE PACKAGE FOR THE LS-GALLERY/ALCOVE PATH MODEL

CRISTIAN LENART

This Maple package implements the LS-gallery/alcove path model for the representation theory of semisimple Lie algebras, that was developed independently in different versions in [1, 3]; see also [4, 5]. This model is a discrete counterpart to the Littelmann path model. It is useful for investigating the combinatorics of irreducible crystals corresponding to the complex semisimple Lie algebras.

The package is based on Stembridge's "coxeter/weyl" packages, available from

`http://www.math.lsa.umich.edu/~jrs`.

Thus, one can work with any irreducible root system. The main procedures in our package are briefly described below. More details and the package itself are available on my webpage

`http://math.albany.edu/math/pers/lenart/articles/soft.html`.

For the terminology used below I refer to [3, 4], or, for a faster access, I refer to the slides available at

`http://math.albany.edu/math/pers/lenart/articles/combin_sli3.pdf`

and the survey available at

`http://math.albany.edu/math/pers/lenart/articles/repres.html`.

Essentially, this model is based on the choice of a sequence of adjacent alcoves joining the fundamental one with the fundamental one translated by $-\lambda$, where λ is a dominant weight corresponding to an irreducible representation of the relevant Lie algebra. Such a sequence, called alcove path, can be encoded by a sequence of roots (corresponding to the walls crossed by the alcove path), which is called a λ-chain. The vertices of the crystal are labeled by certain subsequences of the λ-chain, called admissible subsets; these are defined in terms of saturated chains in the corresponding non-affine Weyl group.

Procedures.

- `prepare` - initializes the basic structures related to the considered root system
- `lchain` - constructs a particular λ-chain of roots
- `fold` - constructs all admissible subsets for a given λ-chain
- `weight` - computes the weight of the representation for an admissible subset
- `admfold` - constructs the folded gallery/alcove path corresponding to an admissible subset
- `tstlrfold` - tests whether an admissible subset indexes an irreducible crystal in the tensor product of two irreducible crystals

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- `fp` - constructs the result of applying a root operator F_p to an admissible subset
- `repf` - applies a sequence of root operators F_p to an admissible subset
- `ep` - constructs the result of applying a root operator E_p to an admissible subset
- `repe` - applies a sequence of root operators E_p to an admissible subset
- `fp2` - constructs the result of applying a root operator F_p to a pair of admissible subsets (in a tensor product of irreducible crystals)
- `ep2` - constructs the result of applying a root operator E_p to a pair of admissible subsets (in a tensor product of irreducible crystals)
- `gototop1` - constructs a sequence of root operators E_p corresponding to a path from a vertex to the highest weight vertex in an irreducible crystal
- `gototop` - constructs a sequence of root operators E_p giving a path from a vertex to the highest weight vertex in a tensor product of two irreducible crystals
- `gotobot1` - constructs a sequence of root operators F_p corresponding to a path from a vertex to the lowest weight vertex in an irreducible crystal
- `gotobot` - constructs a sequence of root operators F_p corresponding to a path from a vertex to the lowest weight vertex in a tensor product of two irreducible crystals
- `compkey` - computes a Weyl group element associated to an admissible subset, which is an analog of the Lascoux-Schützenberger key, and is relevant to the corresponding Demazure modules.
- `sp` - constructs the result of applying a simple reflection to a vertex of an irreducible crystal
- `actw` - constructs the result of applying a sequence of simple reflections to a vertex of an irreducible crystal
- `sp2` - constructs the result of applying a simple reflection to a pair of admissible subsets (in a tensor product of two irreducible crystals)
- `actw2` - constructs the result of applying a sequence of simple reflections to a pair of admissible subsets (in a tensor product of two irreducible crystals)
- `invol` - constructs the result of applying Lusztig's involution to a vertex of an irreducible crystal, see [5]
- `commutator` - realizes the commutator in the category of crystals studied by Henriques and Kamnitzer, see [2].

References

[1] S. Gaussent and P. Littelmann. LS-galleries, the path model and MV-cycles. *Duke Math. J.*, 127:35–88, 2005.

[2] A. Henriques and J. Kamnitzer. Crystals and coboundary categories. *Duke Math. J.*, 132:191–216, 2006.

[3] C. Lenart and A. Postnikov. Affine Weyl groups in K-theory and representation theory. `arXiv:math.RT/0309207`.

[4] C. Lenart and A. Postnikov. A combinatorial model for crystals of Kac-Moody algebras. `arXiv:math.RT/0502147`, to appear in *Trans. Amer. Math. Soc.*

[5] C. Lenart. On the combinatorics of crystal graphs, I. Lusztig's involution. *Adv. Math.*, 211:204-243, 2007.

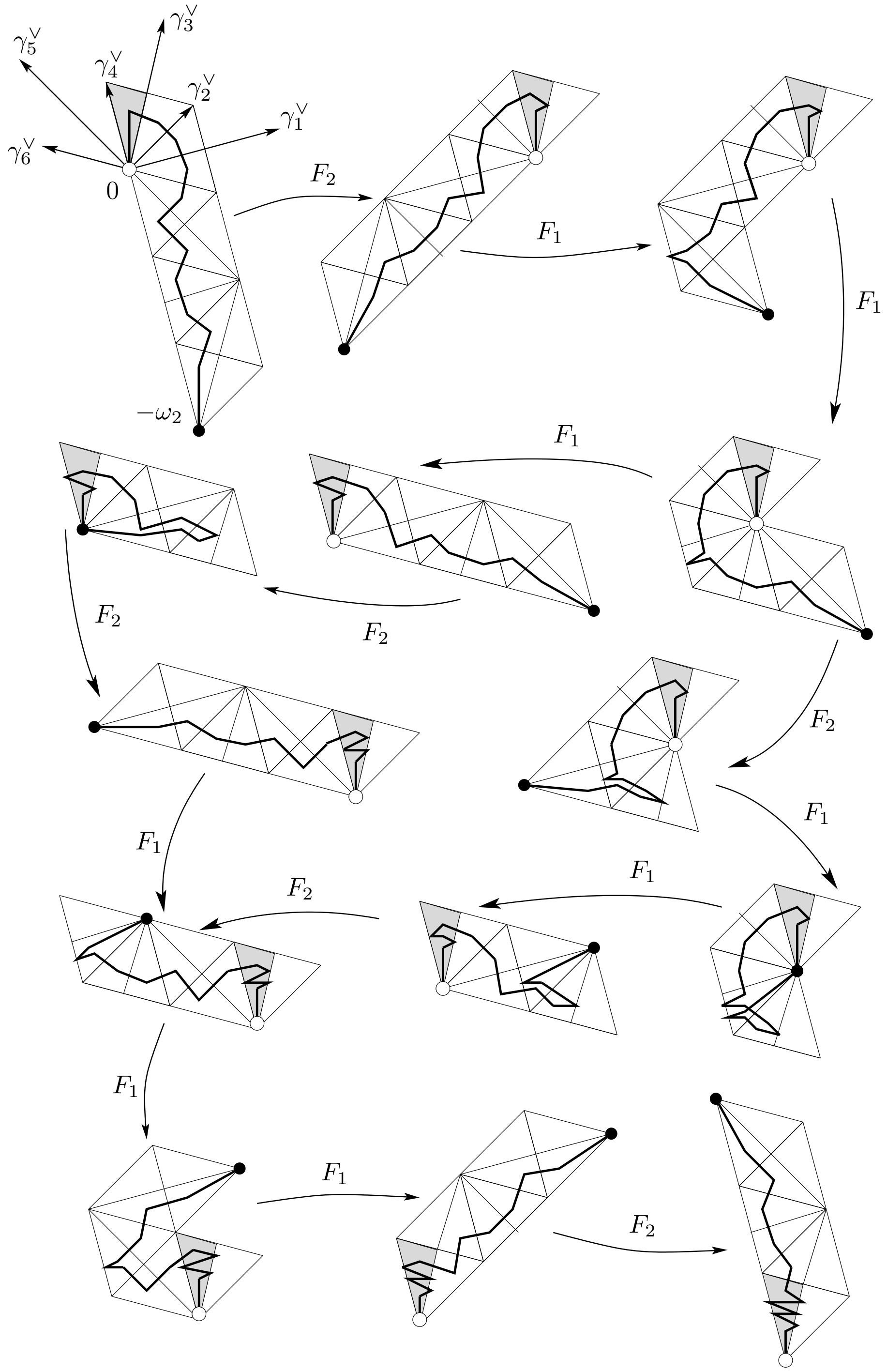

FIGURE 1. The crystal for the fundamental weight ω_2 for type G_2.

DEPARTMENT OF MATHEMATICS AND STATISTICS, STATE UNIVERSITY OF NEW YORK AT ALBANY, ALBANY, NY 12222

E-mail address: lenart@albany.edu